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SDG Target 10.1 Inequity and Inequalities: Measurement Choices and Building Blocks of Poverty Sensitising Indices

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SDG Target 10.1 Inequity and Inequalities: Measurement Choices and Building Blocks of Poverty Sensitising Indices

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Abstract: Widespread inequalities across the economies and within economies are a major policy concern. Over six years of the SDG era, lengthened by the ongoing COVID-19 shock, revealed how harshly inequalities impact on the lowest deciles. A stepping stone for any policy initiative is to have a fair idea of the iniquitous elements across inequalities and dynamic insight mechanisms to ascertain whether these are deepening, and if so at what pace and to what extent. SDG Target 10.1 stresses on the need for a higher income growth of the four bottom deciles, which in turn necessitates tools to assess progress. An analysis of various prevalent indices is carried out in the quest for poverty sensitising (PS) indices. Moreover, some PS indices are put forth including one to evolve an inequity augmented Lorenz curve with the objective to end poverty while reducing inequities. Conclusions and way forward give prescriptions for equitable policies to attain target 10.1 in a lasting manner.

Keywords: Pigou-Dalton transfer, inequality, inequity, poverty sensitising indices, SDGs.

Introduction

The Sustainable Development Goal 10 aims to ‘Reduce inequality within and among countries’. On the question of ‘inequality of what’, there can be many answers. The Target 10.1 specifies growth of incomes as one such parameter. Adopting a normative approach it states: ‘By 2030, progressively achieve and sustain income growth of the bottom 40 per cent of the population at a rate higher than the national average’. The

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pith and substance of this formulation is that the four bottom deciles are not left behind in sharing fruits of income growths vis-à-vis the top six deciles and that alongwith growth the distributional fairness is taken care of. In essence, to achieve the Target 10.1, this enjoins upon reduction of inequalities across all deciles, towards which the parameter opted, could be per capita income or the highly correlated per capita expenditure, and in case of data availability limitations, the parameter values at rather the household level. Still among the unequal per capita incomes/ expenditures the deeper question of measurement of ‘how much unequal’ and what weights a society attaches to the plight of bottom deciles, comprising poor and vulnerable, and also whether these parameters are converging or diverging, still remains to normatively address. This entails choice of indices which effectively capture the poverty, that are termed as ‘poverty sensitising (PS)’ indices in this paper. Such indices can help steer the policies towards assessment of inequalities at the ground level and thus can be well perceived in any endeavour for localization of SDGs.

Of course, all the 17 SDGs are highly interconnected. Among these achievement of SDG 10, of SDG 5 to achieve gender equality and empower all women and girls, and of SDG 1 to end poverty in all its forms everywhere, are highly contingent upon achievement of other SDGs. Normatively speaking, inequalities and poverty are a cause of deep concern, both being so intertwined. Owing to deep interconnects of inequality and poverty, one realises that to seek absolute equality is to ask for too much, and even if once theoretically attained, a society would face the challenge of retaining it. All the more, as witnessed with the advent of COVID-19, it is not necessary that if a society ably reduces its inequalities it can continue so unabated. Not only the pace could decelerate, its direction could also reverse resulting into higher inequalities, which the selected poverty sensitising inequality indices should truly and clearly capture. Undoubtedly during a major shock like the pandemic the poor and unskilled face a higher probability to lose job. Moreover, this compounds the adverse effects created by the flip side of the fourth industrial revolution (IR4.0), which has hit hard against the jobs of repetitive nature, largely occupied by the unskilled

in the informal market, and self-employed in non-resilient livelihoods; both having little job security. Of course, some positive gains of IR4.0 would also trickle down to lower deciles.

In the pursuit of reduction of inequalities, the other SDG 10 Targets encompass to empower and promote the social, economic and political inclusion of all; to ensure equal opportunity and reduce inequalities of outcome; to adopt policies, especially fiscal, wage and social protection; to improve the regulation and monitoring of global financial markets and institutions; to ensure enhanced representation and voice for developing countries in decision-making in global international economic and financial institutions; and to facilitate orderly, safe, regular and responsible migration and mobility of people. In fact many of the SDG 10 targets internalise the enablers, while three specific enablers provided as targets cover the principle of special and differential treatment for developing countries, in particular least developed countries; to encourage official development assistance and financial flows, including foreign direct investment; and to reduce the transaction costs of migrant remittances to less than 3 per cent.

A silver lining on progress of Target 10.1, as enunciated in the 2019 SDG Report of the UN Secretary General is: ‘In more than half of the 92 countries with comparable data during the period 2011–2016, the bottom 40 per cent of the population experienced a growth rate that was higher than the overall national average’. But then the dark clouds of harsh reality convey that the situation wasn’t so converging in the remaining countries among 92 for which at least data availability did allow comparison; apart from the remaining 100 plus SDG signatory countries for which comparisons could not be made for want of data. The Report further points out about less than 25 per cent income/consumption going to the bottom 40 per cent, and voices concerns on the increasing share of income of the top 1 per cent earners. The UN SDG Report 2020 reiterates these facts, pointing that the richest 10 per cent were receiving over 20 per cent of income. Notably, the Report also brings forth that the pre-COVID-19 progress was not on track to meet the Goals by 2030. The UN SDG Report 2021 further adds: ‘The pandemic is likely to

reverse progress made in reducing income inequality since the financial crisis (2007-09).’ The UN SDG Report 2022 points out the first rise in between-country income inequality in a generation.

Globally the inequalities are very sharp, as the Oxfam Report 2019 underscores that in the preceding year 26 richest people owned the same wealth as the 3.8 billion poorest half of humanity. It pointed out increasing inequalities stating that the number of such rich had come down from 43 the year before (Oxfam Report 2019). An Oxfam paper of 2021 argues that billions of people were already living on the edge when the pandemic hit (Oxfam Briefing paper 2021). The Oxfam Policy Paper of 2022 further adds: ‘The wealth of the world’s 10 richest men has doubled since the pandemic began. The incomes of 99% of humanity are worse off because of COVID-19.’

Section 2 of the paper covers review of literature, section 3 discusses Indian inequality and poverty scenario, section 4 is on inequality *per se* and inequity measures, section 5 evolves a normative SDG target 10.1 experiment, section 6 puts forth three poverty sensitising (PS) indices, section 7 evolves inequity augmented Lorenz curve capturing the distribution revealed ratio, as the fourth PS index, section 8 comprises of some inequality *per se* (sans inequity) capturing indicators, and lastly the section 9 crystallizes conclusions and way forward.

2. Review of Literature

Literature in economics is surfeit with the issues of inequality and poverty, their causes, interconnects and policies to meet the challenges. Malthus had argued that the population increases in geometric ratio and outstrips the food supply that increases in arithmetic ratio, which has consequences like possibility of exposure to poverty or charity due to inability to provide educational advantages (Malthus, Thomas 1803). Ricardo, attributing a higher return to better than marginal lands, argued that as wages didn’t partake the proportion going to rent or profits, inequalities increased, adding that inequality in human labour continued over short periods leading to inequality in wages (Ricardo, David 1817). Marx criticised capitalism arguing that it creates class inequalities by bourgeoisie against

proletariat, and advocated: from each according to his ability, and to each according to his need (Marx, Karl 1867). Rawls (1971) argued that given certain assumptions, economic and social inequalities are to be judged in terms of the long-run expectations of the least advantaged social group. He suggested fairness through his prescription of ‘veil of ignorance’ arguing that if one has no idea who one is in the society one would design a fair outcome. Kuznets propounded a hypothesis that as an economy develops, market forces first increase economic inequality and then decrease it, resulting into an inverted letter ‘U’ shaped curve. Taking this work ahead Milanovic argued in his ‘Augmented Kuznets Hypothesis’ that there is another group of factors called the social-choice factors like higher size of social transfers and state sector employment, which lead to reduction in inequality (Milanovic 1994). Acemoglu and Robinson argued that democratisation leads to institutional changes encouraging income redistribution and reduced inequality; but non-democratization associated development may become ‘autocratic disaster’ with higher inequality or ‘East Asian Miracle’ with low inequality (Acemoglu, Daron; Robinson, James, A. 2002). Piketty comprehensively analysed growth of wealth and argued it being faster than that of economic output; and advocated for a global tax on capital to reduce inequalities. Analysing in the context of inequalities across various deciles he added that Gini Coefficient and Theil index are sometimes useful, but that they raise many problems (Piketty, Thomas 2016).

The literature on the marginal utility is also quite relevant to the corrective actions aimed to reduce inequity. It is largely agreed that the marginal utility of any good or service consumed over some period (not allowing its exchange/storage etc.) diminishes as its availability to a consumer increases; assuming that the good (service) is indeed ‘good’ (‘service’) and not ‘bad’ (dis-service) like trash (noise). On this generic strand of micro economics, for instance, Nicholson argues that more quantity of any particular good over some period is preferred to less (Nicholson 1998). This ensures that in the utility-quantity space the marginal utility curve is concave towards the quantity axis customarily selected as the x-axis, and over the portion in which the marginal utility

remains positive. On the utility theory Autor argues that for a risk-averse consumer, the utility of average wealth is greater than the average utility of wealth, so a consumer wants to evenly distribute wealth across states, by equating marginal utility across states (Autor 2019). Further, D. Friedman *et al.* taking recourse to modern Marshallian approach to consumer choice, argued that reformulation of neo-classical marginal utility of money suggested it to be diminishing; and that it can largely supplant as an arbitrary budget constraint under the setting of a partial equilibrium analysis (Friedmann, Daniel; Sakovics, Jozsef 2011). In the economic strand towards equity, Chaturvedi argues that the real challenge lies in making technology as a new equalizer through adequate institutional frameworks, processes and engagements (Chaturvedi 2020).

3. Indian Inequality and Poverty Scenario

Focus of this paper though on the bottom end of the population is not limited to how amidst resource constraints they can be helped better. It goes beyond with the aim that through socio-economic inclusion they can significantly push economic parameters like GDP or per capita income/ expenditure; and contribute significantly to growth, development and sustainability.

3.1 Income and MPCE Inequalities in India

The issue to push a society towards equity, mostly mentioned as equality, is an accepted normative concept, though differences persist on the degree of acceptable level of inequity. Often the idea is to attain a more equal distribution of incomes or consumption or wealth, which in fact, are all highly interconnected. A related issue is that any steps once taken towards equity should not be retracted.

Indian constitution, in its preamble itself enshrines equality of status and opportunity. Under fundamental rights, in Article 14, it guarantees equality before law or the equal protection of laws, within territory of India to all persons, meaning thereby that this protection is not confined only to the Indian citizens but extends to everyone within the physical boundaries of India. The Part IV of the Indian Constitution covers the

directive principles of State policy, which though not enforceable, under Article 38 enjoin upon State to minimise inequalities in income, and endeavour to eliminate inequalities in status, facilities and opportunities, not only amongst individuals but also amongst groups of people residing in different areas or engaged in different vocations. Further, under Article 39 (c) it beacons against concentration of wealth and of means of production to the common detriment (Constitution of India).¹

The canvas of equality and need for reduction of inequalities is very vast, encompassing politico-socio-economic and many other manifestations. To fructify it of course, choices of related parameters need to be looked into.

In India, the NSSO periodically undertakes household consumer expenditure survey capturing monthly per capita expenditure (MPCE) based on recall data of population, as income based data is not available. Decile wise MPCE data gives a broad idea on consumption inequalities. An NSSO study argues that normally, the concept of per capita income – or per capita (overall) expenditure, if income data is not available – is used for comparison of average living standards between countries/ regions (Sarvekshana 2017). In fact, compared to income, consumption is a better parameter to analyse the status of bottom deciles, as it *inter alia* includes transfer payments received as safety nets, like pension schemes, and differential of social assistance availed below market price, which are funded by central government and topped up by the states/ UTs.

The data also indicates the extent to which the inequalities become harsher for a person placed amongst the lower end of the bottom 40 per cent population. Notably, as per **Table 1**, based on the NSSO robust surveys from 1983 to 2011-12, the share of bottom 20 per cent population, in total consumption expenditure remained in single-digit ranging from 8.1 per cent to 9.2 per cent, showing a decline since 1993-94 indicating increased inequality. During the period the share of bottom 40 per cent population, remained 19.6 per cent to 22.3 per cent, which too showed a decline since 1993-94 manifesting higher inequality. Resultantly, within the proportion of the income of the bottom four deciles, the share of its

sub-group of the bottom two deciles hovered narrowly within its 40.5 to 42 per cent.

Table 1 : Share of Various Groups in Total National Consumption Expenditure

NSSO Survey Round	38	50	61	66	68
Reference Period	1983	1993-94	2004-05	2009-10	2011-12
Bottom 20%	9.0	9.2	8.5	8.2	8.1
Bottom 40%	22.2	22.3	20.3	19.9	19.6
Bottom 20% as a per cent of the bottom 40%	40.5	41.3	41.9	41.2	41.3
Bottom 21 st to 40 th percentiles as a per cent of the bottom 40%	59.5	58.7	58.1	58.8	58.7
Proportion of Bottom 21 st to 40 th percentiles to bottom 1 st to 20 th percentiles	1.47	1.42	1.39	1.43	1.42

Source: NSSO Rounds data based authors' computations.

A look at the ratios of shares of the two subgroups implies that the bottom 21st to 40th percentiles had 1.39 to 1.47 times expenditure compared to the bottom 1st to 20th percentiles. This implied that in India, sub-group level total inequalities at these levels continuously existed during the period but were not ultra-harsh. Therefore, it can be safely implied from the Table 1 that when in 2011-12 a high 21.9 per cent population was below the prevalent poverty line as per Tendulkar methodology, the consumption expenditure of the bottom 21.9 to 40 per cent population wasn't extremely better.

The values of Gini coefficient for 2011-12 MPCE for the rural, urban and all India were 28.7, 37.7 and 35.9 per cent respectively. But being a uni-dimensional indicator it doesn't by itself put forth the inter-

decile MPCE inequalities. Moreover, though for a given Lorenz curve, Gini coefficient can be uniquely computed, its reverse is not true, as two different yet crossing-over Lorenz curves may have the same value of Gini coefficient tethered to these. Still the data brings to fore the concern of relatively higher inequality in urban areas, that too amidst inevitable urbanization.

The slope of the end-to-end secant of a Lorenz curve, which is customarily drawn in a unit square to ensure that its height and base are equal, is obviously unity. Corresponding interval of this function $L(x)$ on the x axis is the closed interval $[0,1]$. The preceding Table 1 also throws some light on the lowest segment of the Lorenz curve, which manifests cumulative MPCE shares in the total MPCE. Hence, Table 1 indicates that for 2011-12, from the starting point of the Lorenz curve upto the point at which it covers bottom 20 per cent population (and thus had a base of 0.2), its height was 0.081 (8.1 per cent being the MPCE share of the bottom 20 per cent population as per NSSO 2011-12). This implied that the slope of the secant of the selected bottom segment as 0.405, leading to angle of this secant as 22.048 degrees (or 22 degrees 3 minutes). Now, 'Mean Value Theorem' assures that (at least) one tangent to this segment of the Lorenz curve would also have this slope, and thus at such point of tangency marginal share of the percentile around it would be 0.405 per cent of the total national MPCE. Similarly, for the bottom 40 per cent population (which covers the four bottom deciles), as the income share is 19.6 per cent, the slope of the secant would be 0.490 or 26.105 degrees (26 degrees 6 minutes), assuring that (at least) one tangent to this larger lower part of the Lorenz curve would have this slope, thus at such a point of tangency marginal share of the percentile around it would be 0.490 per cent of the total national MPCE.

After 2011-12, as no NSO robust survey results are available, to discern the Indian situation just prior to SDG era, one may look at the 'Economic Survey 2015-16'. It pointed out income inequalities in many countries, with concentration at the top; and that in 2013-14 in India, the top 1 per cent, 0.5 per cent and 0.1 per cent accounted for a high 12.4 per cent, 9.4 per cent and 5.0 per cent respectively of the total income.

It added that such a concentration was comparable to the one in the UK, though lower than in the USA (Economic Survey 2015-16)². On the trend of increasing concentration, it indicated a higher increase in income concentration in India compared to the UK and the USA.

Moving from income to wealth inequality in India, the 2018 Oxfam report placed the Gini coefficient of wealth at 0.83 in 2017 the preceding year. This indicates much deeper inequalities compared to in incomes. Obviously, with the causality running both ways, these leapfrog with income inequalities. World Inequality data for India similarly indicated that the share of bottom 50 per cent adults in pre-tax national income, had increased from 20.5 percent in 1951 to 23.6 per cent by 1982, but declined to 13.1 per cent by 2019.³

Notably, economic poverty line is linked to minimum absolute income level for a person/a household, and doesn't directly take into account concentration of incomes. In the setting of SDGs, now economic as well as multidimensional poverty is captured through a set of national/global indicators.

4: Inequality *per se* and Inequity Measures

4.1 Inequality *per se* and Inequity

The literature treats desirability for equity on normative basis. Most of the economic literature uses the word 'inequality' to denote both 'inequality *per se*' and 'inequity'. Paper later covers 'inequality *per se sans* inequity, and discusses some indices that can capture it.

In real life situations unequal economic distributions are encountered due to differential levels of assets both physical and human capital, rent seeking, asymmetric market information, vectors of circumstances as well as of efforts and so on. A look at the underlying causes of inequity points out at the inheritance structures of wealth and family knowledge base; unequal opportunities to acquire education, learning and skilling; inadequate capital base to access credit and undertake entrepreneurial initiatives; social and cultural norms; besides the factors like differences in own effort levels once the adversities owing to circumstances are duly accounted and compensated for.

In the literature related to inequality, there are a number of principles on which various indices/ measurements are analysed, of which the four prominently used can be expressed as follows.

4.2 Inequality Indices and Principles to Hold

Anonymity principle: It does not matter who earns the income, or for any of the possible permutation of income. Thus the anonymity implies to account for only the ordering in ascending (precisely non-decreasing) order ignoring the identity, and resultantly two identical distributions should give the same index.

Population principle: If an income distribution is exactly repeated ('cloned') doubling its size, the inequality measured should remain unchanged.

Relative income principle: Only the relative incomes should matter, not their absolute values. In fact, as the marginal utility of income diminishes, an index based on relative incomes is better than the one based on absolute incomes, as it entails a rise in the same percentage across all incomes to keep the value of the index unchanged.

Pigou-Dalton principle: It necessitates in essence that if one income distribution can be achieved from a second one through a sequence of mean preserving regressive transfers, then the first one happens to be more unequal. It is a principle of welfare economics in cardinal setting, rooted *ceteris paribus* in that a progressive transfer of some variable (say, utility or income or expenditure or wealth) from the rich to the poor is desirable, if it does not make a rich transferor poorer than the transferee poor. Stated opposite way, a regressive transfer is one from a relatively poor (not richer) to a relatively rich (not poorer) individual, thus both essentially remaining on their respective original side of the mean income. Thus, if originally one poor has an income 'a' while a rich an income 'c', such that $a < AM < c$, where AM is the mean (arithmetic) income; after a regressive transfer 'k' (> 0 but $< a$) from poor to rich their incomes would become $(a-k)$ and $(c+k)$ respectively, and this regressive transfer should lead to a higher value of the inequality index.⁴

In practice, in no way this principle supports any regressive transfers but genesis of its wording is rank preserving.

Obviously, value of any indicator worth its salt should collapse to zero if all the terms are equal manifesting not even a shred of inequality.

Some prominent measures of inequality are covered next.

4.3 Lorenz Curve and Gini Coefficient

A Lorenz curve throws certain insights into inequalities, and is drawn after arranging incomes (or for that matter any other variable for which inequality is to be captured), in an ascending (to be precise non-descending) order. It depicts cumulative share of population (or households) on the horizontal axis and cumulative share of incomes on the vertical axis, using unit base and height in the cumulative-population cumulative-income shares space.

For ordering the incomes, even if entire population data is not available but percentile data is available, the same can be arranged in ascending⁵ order of share of total income. Due to such ordering, the slope of a Lorenz curve can never become flatter while moving towards higher cumulative shares. Resultantly, the slope of the secant joining the origin to the varying increasing cumulative share points on the curve, would also never diminish.

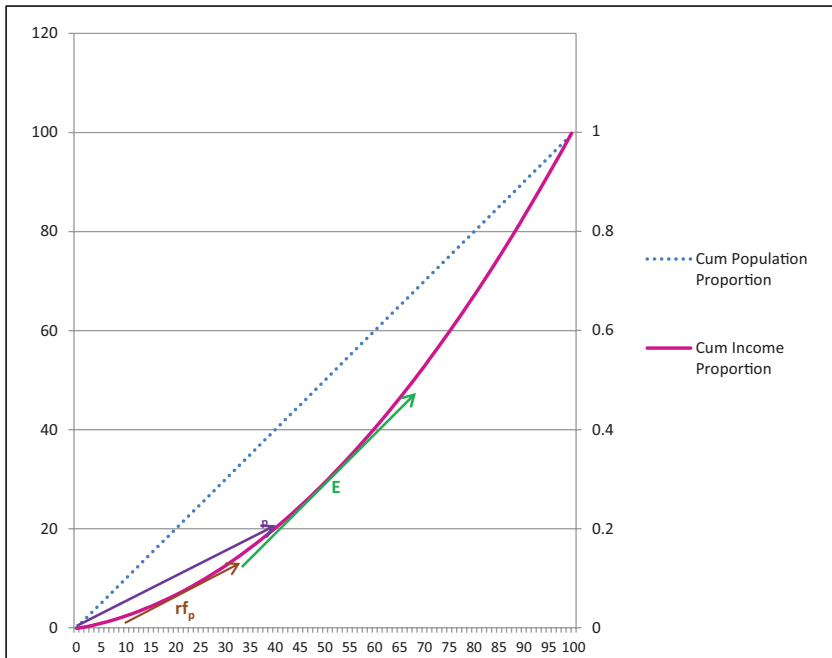
The Gini coefficient represents the proportion of area of the lens (plano-convex being plain along 45 degree line and convex towards primary x axis and secondary y axis) between the Lorenz curve and the 45 degrees line, to the total area of the triangle under this line. So there can be a situation that any two Lorenz curves that cross over may have identical Gini coefficients if inequalities in their different portions exactly cancel out. Of course, in case one Lorenz curve remains invariably closer to the 45 degree line compared to the other one, being a case of 'Lorenz Dominance', they would never cross, and the first one would invariably show lesser level of inequality, and so higher level of total welfare. The precise formulation is defined that cumulative distribution F_h weakly Lorenz dominates cumulative distribution F_g if and only if $L_h(t) \leq L_g(t)$

for any $t \in [0; 1]$, where $L_g(t)$ and $L_h(t)$ are Lorenz curves for income distributions $F_g(x)$ and $F_h(x)$.

Moreover, as the Lorenz curve is a continuous function on the closed interval $[0,1]$, and differentiable on the open interval $(0,1)$, the ‘mean value theorem’ states that there exists a point in $(0,1)$ such that its slope is also unity. In case the mpce was monotonically ascending, the slope would have increased monotonically. Honouring the non-descending condition of ordering, one can still say that the slope can either increase monotonically, or for any intermittent intervals of equal mpce, remain constant. A single inequality statistic like Gini coefficient may capture average but not different parts of distribution (Cigano 2014).

The Lorenz curve (**Figure 1**) scores over the Gini coefficient in revealing more information in several ways as listed next.

Figure 1: Lorenz Curve and Tangents at Points of Mean Income and its Half (based on the SDG 10.1 experiment)



Source: Authors' compilation.

- i. Dynamic characteristics of a Lorenz curve, like how slopes of its tangent and secant change throw additional insights. The point 'E' at which the tangent to the Lorenz curve has a slope of 45 degrees, is the point at which share of marginal population equals share of marginal income, which happens to be AM of income.
- ii. Thus the percentile around it has one per cent share of total income.
- iii. It is the precise transition point indicating that below (above) it the share of each percentile in total income, is less (more) than one percent. Similar exercises can be undertaken for other specific proportions of the total income.
- iv. Another analysis can manifest a representative percentile of a bottom portion of the curve. For instance we mark the point ' f_p ' on the Lorenz curve such that 'curve Of_p ' covers the bottom 40 per cent population part of the curve having say b per cent of total income (where b is obviously less than 40, and the letter b is selected to denote bottom population), and draw the straight line secant ' Of_p ' from the origin 'O'. Now, the point rf_p is marked as the tangential point having the same slope as of the secant ' Of_p ' drawn. Value of the slope of such tangent would thus be representative of the population in the bottom forty percentiles, and to be precise its slope at rf_p (denoting representative of forty percentiles) would represent the average income of such forty percentiles as a proportion of the average of total income.
- v. The Lorenz curve also spurs Hoover index, another inequality measure sometimes termed as Robin Hood index, defined as the maximum vertical gap between it and the egalitarian line. This gap crystallizes the proportion of total income, that all sub-AM earners are cumulatively short of to reach egalitarian line, or $\sum (AM - x_i)$ for all $x_i < AM$.
- vi. Building upon the Lorenz curve, Park and Kim argue that the value of an individual's welfare starts rising with income but not rapidly, till the income to support basic living.⁶ Thereafter, it rises rapidly as individual feels economic freedom, but it again slows down due

to saturation. Accordingly, the welfare function for an individual 'i' can be expressed as a sigmoid function,

$U(y_i) = [1 / \{1 + e^{-\alpha(\mu - y_i)}\}]$, where μ and α are constants, and further the social welfare function is the summation over all the individuals. They next argue that the feasible income equality is the optimal income distribution that maximizes total social welfare, without hampering sustainable economic growth of the society. The reasons adduced by them for inequality among feasible equality curves across countries, of course needs more research.

- vii. It can be perceived that in the case of equal Gini coefficients emanating from two intersecting Lorenz curves the one which is initially lower indicates higher inequalities inflicted upon lower percentiles, upto the percentile of intersection point.
- viii. Accordingly, for research into inequalities among bottom 40 per cent, the point of slope of tangent proves to be a much better parameter than Gini coefficient.

In the context of SDG Target 10.1 it would thus be critical to utilise Lorenz curve to inter-temporally analyse:

- i. Whether the rate of growth of cumulative income of the bottom 40 percentiles exceeds/equals/falls short of the rate of growth of cumulative income of the entire population, i.e. all the 100 percentiles.
- ii. Consequently the share of income of the bottom 40 percentiles in total income increases/remains constant/diminishes respectively.
- iii. Moreover, if at the beginning of the SDG era the initial share of income of the bottom 40 percentiles was say, 20 percent of the total income, a nation may strive to increase it by, say, 5 per cent points to 25 per cent by 2030. The aspired Lorenz curve by definition should shrink towards the egalitarian line, unless strong top loaded concentration crops up among the top six deciles.
- iv. The tangent points where slope to Lorenz curve is half or one-fourth would represent the percentiles for which marginal incomes are half or one-fourth of the arithmetic mean (AM) income. At these points tangents would form angles of approximately 27 degrees or 14 degrees respectively to horizontal axis.⁷

The enormity of challenge of SDG Target 10.1 to progressively achieve and also sustain income growth of the bottom 40 per cent of the population at a rate higher than the national average, needs to be analyzed in the light of status and dynamics of the share of the bottom 40 per cent and 20 percent population over the years as given in Table 1, which indicates a fall since at least since 1993-94. Therefore, the task at hand is trend reversal followed by its sustenance. It is a fact that at the time of setting the Target 10.1 no minimal income growth differential in favour of these deciles was set, therefore, sooner than later this silence needs to be broken, addressed, fructified and sustained to reduce inequalities to a socially acceptable level.

4.4 Arithmetic Mean (AM) Anchored Indicators

In this paper, we are using terms rich or poor in the relative sense for the persons/ households above or below AM or GM (Geometric mean). We are thus overlooking the possibilities that in absolute terms a household termed rich may actually be poor, since a lopsided distribution of a very small cake leaves everyone poor, though in varying degrees. On the other hand, a rich household may be ultra-rich, all other households being just plain vanilla rich! Thus the nomenclature ‘rich’ or ‘poor’ used here for the households (or persons) is just for the sake of distinction in relative terms compared to AM (or latter on to GM).

In a wide class of indices the Arithmetic Mean (AM) is selected as the anchor to further build upon. So far so good, however, the sum of deviations from AM (by definition) adds up to zero. Literature is surfeit with the measurement techniques based on squaring up of deviations to convert the negative values also into positive ones. The resultant ‘statistic’/ parameters, also termed as the second moment of deviations lead to Variance, Standard Deviation (SD), and Coefficient of Variation (CV) etc.

Notably, the CV, i.e. SD/AM is computed through division of positive value of SD, by the absolute value of the AM to keep it positive. It isn’t defined when AM is zero, unless both AM and SD being zero its limit can still be computed. CV helps to compare variability across data

sets that widely differ in absolute values, say, incomes in a developed country vis-à-vis in a less developed country. However, one disadvantage of CV is that it becomes very sensitive as AM tends to zero, say, while comparing household savings in a less developed country, when dis-savings are assigned negative values.

Another shortcoming of such second moments like variance, SD, CV is that on squaring, the data entries located farther from the AM get a higher Weightage. For instance if the variable takes five values, namely 10, 20, 30, 40 and 50, the mean being 30 the highest and lowest entries contribute 400 each to the sum of squares, while the two closer (yet distinct from the mean) contribute 100 each; of course the middle one rightly contributing nothing, being the mean itself. In this case in spite of subsequent operations of division by mean (in case of population or by mean minus one in case of a sample), followed by taking a square root the impact of each of the extreme entries remains 4-fold of each those closer to (yet distinct from) mean.

Compared to the SD, the Mean Absolute Deviation (MAD) captures deviation, without any distance-from-mean related amplification. MAD, measuring absolute value of deviation from AM, gives an idea of dispersion. However, when all the terms are doubled, so does the MAD. Therefore, it is linked to the absolute value (of incomes/ MPCEs) and thus doesn't capture relative values, rendering it unfit for comparison purposes, discounting its otherwise usefulness.

A basic feature of parameters like variance, SD and CV is that if for a population, each term is doubled, so is the mean, and also distance from the mean. As a result the contributions to variance being squared distance from mean become four times, resulting into a four-fold variance, as shown next.

Mathematically, if the original N population terms are $x_1, x_2, x_3, x_4, x_5, x_6 \dots x_N$, having AM as the mean, on doubling these become new N terms $2*x_1, 2*x_2, 2*x_3, 2*x_4, 2*x_5, 2*x_6 \dots 2*x_N$ with $2*AM$ as the new mean. The new population variance is $(1/N) \sum (2x_i - 2*AM)^2$, or, $4*(1/N) \sum (x_i - AM)^2$, or $4*$ original population variance.

In such a case the standard deviation (SD) is doubled. Therefore, neither variance nor SD clears the ‘relative income principle’. Resultantly, the issue of whether a biased or an unbiased estimator of SD should be used need not be looked into.⁸ As an aside, whereas variance and in turn SD fulfill the anonymity principle, these clear the population principle only for the biased estimators but fail to do so for the unbiased estimators.

Remarkably, the dimension-free parameter CV fulfills the anonymity and relative income principles for both biased and unbiased estimators, and this lends it the characteristic to compare two widely differing income distributions. It clears the population principle for biased estimator but fails so for the unbiased estimator. It also clears the Pigou-Dalton criterion for biased estimator, because in the case of a mean preserving regressive transfer from a poor to a rich the variance increases.

Notably, though the indicators Variance, SD and CV fulfill the Pigou-Dalton transfer principle, yet, contributions to each of these from a ‘poor’ (income below AM) and a ‘rich’ (income above AM) is alike, if both are equidistant from the AM. Therefore, these indicators are indifferent towards inequity and basically capture only the ‘inequality *per se*’.

4.5 The Family of Theil Indices

A number of ‘statistic’ like Theil-T (or T_T) and Theil-L (or T_L) are extensively used to measure inequality. These two indices are in fact special cases of the general entropy index $E(\alpha)$, which is expressed as:

$$E(\alpha) = [1/\{N*(\alpha^2 - \alpha)\}] * [\sum \{(x_i / \bar{x})^\alpha - 1\}]$$

Here, subscript ‘i’ varies from 1 to N, and \bar{x} is the notation for the AM. Now as the parameter alpha approaches 0 or 1, both the numerator and denominator tend to zero throwing up indeterminate forms. Therefore, application of L’Hopital’s Rule⁹ is resorted to in conjunction with the Taylor’s expansion, and in these cases the first derivatives make the ratio determinate (FAO- Bellù, Lorenzo Giovanni; Liberati, Paolo 2006).

When alpha equals zero, the ratio is called as Theil-L and expressed as:

$E(0) = [1/(N)] * \sum [\ln (\bar{x} / x_i)]$ (the summation being over all the i values).

Where, \bar{x} = AM (arithmetic mean). In fact, Theil-L index is anchored to GM (geometric mean) also besides AM, as it can be expressed as the \ln of N^{th} root of $\{(AM)^N / (GM)^N\}$, or, $\ln \{(AM) / (GM)\}$; its genesis emanating from the fact that because AM exceeds GM, when any inequality exists, their ratio exceeds unity and its \ln is positive.

Or on tossing the argument of ‘ \ln ’ and thus putting the negative sign outside, it is alternatively expressed as:

$E(0) = [-1/(N)] * \sum \ln (x_i / \bar{x})$ and as the expression suggests, it is also the Mean Log Deviation (MLD), composed of the average value of $(\ln x_i - \ln \bar{x})$, albeit with a negative sign.

When alpha equals one, the ratio is called Theil-T index and can be expressed as:

$E(1) = [1/(N)] * \sum [(x_i / \bar{x}) * \{\ln (x_i / \bar{x})\}]$ (the summation being over all the ‘ i ’ values).

Theil’s measure may also be expressed as $T = \ln(n) - S$,

where S is the Shannon entropy or information content of the distribution, which has a range from 0 to $\ln(n)$ with n being number of partitioned groups of the population; moreover, Theil-T and Lorenz curve are also interlinked (Rhode, Nicholas 2007).

Shape of the Lorenz curve gives an impression of a higher emphasis to the terms closer to the average population, i.e. the wider widths of the ‘lens’, and in the process the lowest deciles (as well as the highest ones) seem to make relatively smaller contributions towards the curve drawn and in turn the Gini coefficient. However, by construction its starting and end points need to lie on the egalitarian line and what it manifests is on the cumulative basis.

4.6 Atkinson Index

On the diminishing marginal utility of income Easterlin argues that few generalizations in the social sciences enjoy wide-ranging support like it, adding that the effect of a \$ 1,000 increase in real income on subjective

well-being becomes smaller for the higher initial income level (Easterlin 2005).

The strand of diminishing marginal utility of income helps formulate the Atkinson's Index used to measure inequalities. Atkinson argued the concept of Equally Distributed Equivalent (EDE) income. The crux of the Atkinson's inequality index is that EDE is such a level of income that, if obtained by every individual, doesn't exhaust the entire income, though keeping the overall utility intact. Alternatively, if all income is completely and equally distributed, it puts the society at a higher total utility, and therefore a higher welfare level.

The Atkinson index is expressed as:

$[1 - \{(y_{ede})/\bar{y}\}]$ in this notation, y_{ede} is the Equally Distributed Equivalent Income, \bar{y} is the AM, whereas y_{ede} is computed using ϵ the parameter of inequality aversion as:

$$y_{ede} = [(1/N) * \sum y_i^{(1-\epsilon)}]^{1/(1-\epsilon)} \text{ for } \epsilon \neq 1$$

$$\text{and } y_{ede} = \prod (y_i)^{(1/N)} \text{ for } \epsilon = 1$$

The form of the utility function argued by Atkinson, for the i^{th} individual is,

$$U(y_i) = \{(1/(1-\epsilon))\} * y_i^{(1-\epsilon)} \text{ for } \epsilon \neq 1$$

$$\text{and } U(y_i) = \ln(y_i) \text{ for } \epsilon = 1$$

Finally, as mentioned earlier, the Atkinson Inequality Index is expressed as:

$$[1 - \{(y_{ede})/\bar{y}\}].$$

In essence taking advantage of the fact that marginal utility of income is diminishing, an egalitarian society may maintain the existing level of total utility by equally redistributing part of incomes to maintain average utility, and thus leaving some surplus income; or completely and equally distribute income and raise average utility level. In fact, this holds for many other indices also.

5. A Normative SDG Target 10.1 Experiment

In line with the SDG Target 10.1 the Paper carries out a normative analytical experiment to assess how the incomes of the four lowest deciles can rise to catch up with the rest. In this experiment an assumed population

initially has its lowest percentile having a uniform per capita income of say, 100 units, and across the percentiles the incomes monotonically and uniformly increase to 1,000 units for the highest percentile, indicating a 10:1 ratio between the highest and the lowest incomes. Obviously the incomes arranged from the lowest to the highest percentile are non-decreasing (in fact, in this case monotonically increasing).

Now to add the element of growth, starting the initial situation the incomes of all the percentiles are assumed to increase in such a manner so as to start converging. For instance, while the annual growth rate of income of the lowest percentile is taken as 6 per cent, it is set to uniformly diminish across the higher percentiles reaching down to 3 per cent for the highest income percentile. Further it is assumed that the society sustains these differential and converging income growth rates unabated for a reasonable period of say, 5 years in a row.

This little normative experiment can no doubt be expanded to a full blown simulation by varying the parameters involved. Nevertheless with the parameters assumed the distribution and its change over 5-year period indicate:

- i. The Gini coefficient of the initial Lorenz curve is 0.275 which diminishes to 0.257 for the new curve indicating reduced concentration of incomes. Similarly, Theil–T index diminishes from 0.124 to 0.108 after 5 years, and the Theil L starting at 0.150 diminishes to 0.131 after 5 years.
- ii. As the incomes start converging, the ratio of the highest income to the lowest income falls from 10.00 to 8.66, a desirable transition.
- iii. Share of income of the four lowest deciles, i.e. the bottom 40 per cent population, rises from 20.17 per cent to 21.34 per cent. As an aside though incomes rise across all percentiles, the share of income rises up to the bottom 63rd percentiles and falls thereafter.
- iv. An inter-percentile comparison indicates that while initially the average of income of the bottom 20th and 21st percentiles represented average income of the bottom 40 percentiles, on transition the new average of these two percentiles exceeds the latter (the new average income of the bottom 40 percentiles) by 0.6 per cent.

- v. Moreover, such lowest percentile that has more than half per cent of total income shifts from the 21st to 19th percentile, a small yet favorable change. Similarly, the lowest percentile having more than one per cent of total income shifts from the 51st to 50th percentile.
- vi. The slope of tangent to Lorenz curve at the bottom 40th percentile rises in the new curve by almost one degree¹⁰ indicating an increased marginal income share.

Now, reverting to concavity of utility function one perceives some scope to impose some tax on rich and grant some subsidy to poor, of course short of swapping their positions. The concavity also gives an indication of a theoretical equal income to each household that keeps total utility intact, while sparing some fraction of it. This entails to set a choice on the exact shape of the concave curve, at least for the interval covering the household incomes. A simpler way could be to have a rule of thumb broadly acceptable to the society, say, that when income becomes 5-fold utility rises 3-fold, which can of course be modified as per local perceptions. This in the realm of a power function gives $U(I) = c * I^k$ and similarly, $U(5I) = c * 5^k * I^k$ leading to $5^k = 3$, where $U(I)$ is the utility function of income I , c is a constant and k a power such that $0 < k < 1$.

Now, $dU/dk = c * k * I^{(k-1)}$ is positive, as k is set positive, and $d^2U/dk^2 = c * k * (k-1) * I^{(k-2)}$ is negative, due to $(k-1)$ set as negative. Further, as $5^k = 3$ leads to $k = 0.682$, therefore, in this case for the sake of simplicity one can assume $k = (2/3)$ a close by value. The power utility function has an edge over the logarithmic utility function. With $k=(2/3)$ each ten-fold income led utility is $(10)^{0.6667}$ or 4.64 times. Alternatively, for a (natural) logarithmic utility function each ten-fold income increase led utility rises by 2.30; here the change being only additive compared to multiplicative in the preceding case. In the SDG Target 10.1 experiment for the three incomes 100 and 500 and 1,000 the $k= 2/3$ power function gives utility values as 21.54, 63.00 and 100 respectively, being in the ratios of 1: 2.92: 4.64. Alternatively, the \ln operator gives utility values as 4.605, and 6.215 and 6.908 respectively, being in the ratios of 1: 1.35: 1.50, showing much slower relative increases.

6. Some Poverty Sensitising (PS) Indices

6.1 A PS Building Block

Equity demands that to measure it cardinally, the very index utilized should itself be equitable. To ensure this on *ab initio* basis, one way is to accord a higher weightage to the contribution coming from the x_i terms below arithmetic mean (AM), than above it.

One such building block can be,

$\{(AM - x_i)/ (AM + x_i)\}$ because as the term x_i rises from zero towards AM, its contribution remaining positive monotonically falls from unity towards zero, and on x_i reaching exactly equal to AM, its contribution becomes zero. To have a feel of its values for negative values of x_i let us rearrange it as:

$$[-1 + \{(2*AM)/ (x_i+AM)\}]$$

$$\text{Or, } [-1 + \{(2)/ \{(x_i/AM)+1\}\}].$$

Therefore, its first part being minus one, a constant, and the second part diminishing monotonically towards zero as x_i increases, the overall expression tends to minus one. In fact, differentiation w.r.t. x_i at the second last step, yields,

$2*AM * [(-1)* \{(x_i+AM)^2\}]$, which being nothing but $-2*AM$ divided by a positive number never becomes zero.

Therefore, the fall of $\{(AM - x_i)/ (AM + x_i)\}$ is monotonic.

Hence, as x_i rises further beyond AM, its contribution becomes negative and keeps algebraically falling and tending towards (-) 1. This building block thus becomes a stepping stone for the first generic family of PS indicators selected as follows.

6.2 PS Generic One Family of Indicators

With the above background we may try the family of indices having a generic form of the ϕ^{th} power of $\{(AM-x_i)/(AM+ x_i)\}$ averaged over the N terms, with ϕ set as positive but not restricted to natural numbers;

$$\text{or } (1/N) \sum \{(AM-x_i)/(AM+x_i)\}^\phi,$$

The first indicator (PS1) in this family by setting $\phi = 1$ is thus,

PS1 = (1/N) $\sum \{(AM-x_i) / (AM+x_i)\}$. We would modify it by taking absolute values of each contribution and call as PS1B.

Further, by setting $\phi = 2$ we get PS2 as indicator,

PS2 = (1/N) $[\sum \{(AM-x_i) / (AM+x_i)\}]^2$, and positive square root of PS2 as PS2B.

As discussed, a look at the first member of this family indicates that while the contributions to the index, of x_i values $<AM$ are positive, these become negative for $x_i > AM$. Similar is the situation for any other odd-power bearing family member $\{(AM-x_i) / (AM+x_i)\}^{(2m+1)}$, where m is any natural number. For the even-power bearing members $\{(AM-x_i) / (AM+x_i)\}^{(2m)}$, the contributions are nevertheless positive from both sides of AM .

To have a feel of the extreme values of PS1, the first case can be of extreme concentration with the $(N-1)$ households having zero income (surviving on MPCE through transfer/ safety nets etc.) and the sole rich resultantly having $N*AM$ as its income. The index thus becomes,

$$(1/N) * [(N-1)*1 + \{(AM- N*AM) / (AM + N*AM)\}],$$

$$\text{or, } (1/N) * (N-1)* [1 - \{1/(1+N)\}],$$

$$\text{or, } \{(N-1) / (N+1)\},$$

It is less than one, though in the limiting case of a large sample/population, i.e. when $N \gg 1$ it tends to one from the lower side {technically, to set the value of extreme inequality as one, the PS1 index needs a multiplicative correction factor of $\{(N+1)/(N-1)\}$.

Some of the characteristics of this index are:

- i. In the case of extreme inequality i.e. total concentration of income (or wealth etc.) it tends to 1 for a large sample/ population.
- ii. In the case of an egalitarian society i.e. each $x_i = 1*AM$, each term contributes zero, so the index becomes zero, as desired of any inequality index when there is no inequality.
- iii. Therefore, it is not open-ended but is automatically normalised as it varies between zero to one.

- iv. It holds the principles of anonymity, relative income, and population by construction.
- v. Let us try a Pigou-Dalton transfer k from $c > AM$ to $a < AM$ such that the two final incomes follow $(a+k) < AM < (c-k)$, while AM is preserved

To start with, the contribution of any x_i to the index $(AM-x_i)/(AM+x_i)$ can be rewritten as $[1 - \{2x_i / (AM + x_i)\}]$,(1)

As an example let us ascertain the impact of transfer of an amount $k = 0.1 * AM$, on the value of the index. If for one poor x_i increases by $0.1 * AM$, on transfer from one rich whose income falls by $0.1 * AM$, the impact on $\{2x_i / (AM + x_i)\}$, is higher on the smaller number or for a poor compared to a rich. Therefore, as a net impact there is a fall in the value of the inequality index, holding Pigou-Dalton principle (technically worded as a regressive transfer from a poor to a rich leading to increase in the inequality index).

Alternatively, as already calculated $\partial / \partial x_i$ the partial differentiation of (1) has a value,

$(-2 * AM) * \{1 / (AM + x_i)^2\}$. Therefore, it being negative, for a poor on receiving a positive transfer, his contribution to the index falls. On the contrary, for a rich on fall of x_i on making a transfer, the contribution to the index algebraically rises, but by a smaller magnitude, due to a relatively larger denominator. In totality the index falls on such a transfer from a rich to a poor.

A more formal treatment, computes the exact change (final value minus original value) as:

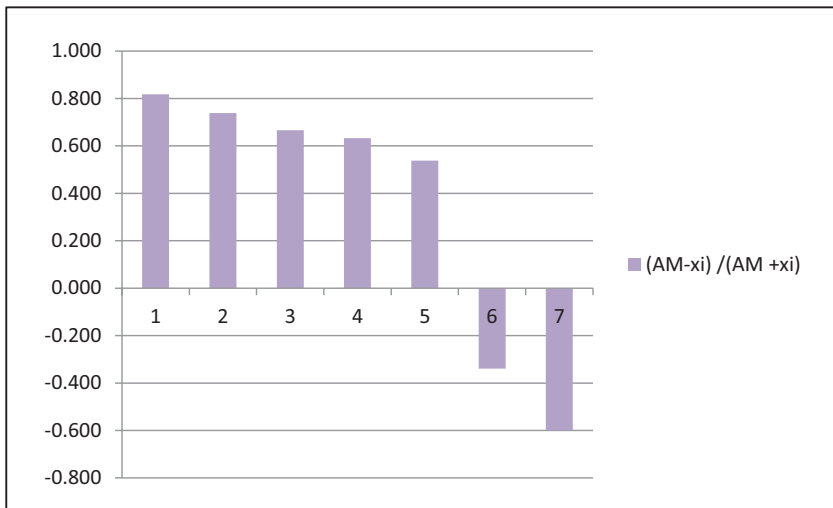
$$(-2) * k * AM * [\{1 / (AM+a)\} * \{1 / (AM+a+k)\} - \{1 / (AM+c)\} * \{1 / (AM+c-k)\}],$$

A comparison of the two terms within the large bracket reveals that each denominator of the first term is smaller than each denominator of the second term, $(AM+a)$ being $< (AM+c)$, and $(AM+a+k)$ being $< (AM+c-k)$, rendering the net term in the large bracket positive.

Therefore, the overall expression is negative. This indicates a fall in this inequality index in case of a transfer from a rich to a poor. This implies that the Pigou-Dalton criterion is fulfilled.

We may have a feel of the index from an example as depicted in the **Figure 2**. Here the seven income values as expressed in terms of AM are 0.1, 0.15, 0.2, 0.225, 0.3, 2.025 and 4.0 averaging, as expected, 1. These terms on computations lead to the $\{(AM-x_i)/(AM+x_i)\}$ values as approx. 0.818, 0.739, 0.667, 0.633, 0.538, (-) 0.339 and (-) 0.600, as shown in the Figure, leading to its average 0.351 as the value of index PS1 in this example.

Figure 2: Heights Showing Contributions $(AM-x_i)/(AM+x_i)$ to PS1



Authors' compilation.

6.3 PS1 Index: A Poverty Sensitising Trigonometric Manifestation

The fact that the contribution of each term to PS1 is within the closed interval (-) 1 to (+) 1 gives an idea to evolve a pictographic presentation in which the value of tangent of an angle represents its contribution. To ensure so, the angle is varied only from (-) 45 degrees to (+) 45 degrees. Accordingly, **Figure 3** is evolved in which a circle is drawn

An Example							In essence	
Point	Income (in AM terms)	(AM - Income)	(AM + Income)	Contribution to index is ratio (AM - Income) / (AM + Income)	Expressible as			
					Ratio	Tangent of angle	Angle in degrees	
A	0	1	1	1	AP/OP	AOC	45	PA
B	0.5	0.5	1.5	1/3	BB'/OB'	BOC	18	PB''
C	1	0	2	0	C to C i.e. zero / OC	COC	0	PP i.e. zero
D	2.5	(-1.5)	3.5	(-) 3/7	DD'/OD'	DOC	(-)23	PD''

Note: 1. O is the origin, with OX as X axis and OY as Y axis. A circle is drawn with the centre P, and radius OP which equals Arithmetic Mean (AM) for normalisation. At the origin O the Y axis is tangential to the circle.

2. All households' incomes are normalised in AM terms, and of the 4 households A,B,C and D are shown above, averaging 1*AM.

3. Contribution of each income to the index is set within +1 and (-)1, being +1 when income is 0 at A and (-) 1 for an extremely high income. Locus of all possible household incomes forms the slide AW with the poorest at the top (for a change) at A as its contribution is +1, which monotonically slides down along the line AW towards (-)1 for an income extremely high, like the two rail lines, angle XOW asymptotically approaches angle XOV i.e. (-) 45 degrees, e.g. for income = 100*AM, contribution is (-) 99/101 or (-)0.9802 and angle XOW becomes (-)44.43 degrees. The last Column represents contributions to index shown on the vertical diameter APV'' ranging from 1 to (-) 1.

4. Finally, value of this poverty sensitising index1 (PS1) is the average over all contributions.

with coordinates of its centre ‘P’ as (AM, 0), the origin ‘O’ such that OY is the Y axis, perpendicular to line OP the X axis. Incomes of all the ‘N’ households are normalised by expressing in terms of AM. Inherent idea is to depict contribution for income x_i to PS1 {which is $(AM-x_i)/(AM+x_i)$ }, as height $(AM-x_i)$ and base $(AM+x_i)$ in a right-angled triangle, both starting from origin O. Accordingly, for a random household H_i plotted with income x_i , the base is automatically $(AM+x_i)$ and height is $(AM-x_i)$.

The poorest possible household having zero income is depicted as at point ‘A’ on the circumference, and thus the angle POA is 45 degrees by construction and value of its tangent being (+)1 gives contribution of such poor to the index (before division by N) as 1. Each household is represented as a dot on the solid sliding line AW, capturing it by assigning a height $(AM-x_i)$ and with its base on X axis. Base is measured from origin O, and thus for all $x_i \leq AM$ the base $(AM+x_i)$ is somewhere on the right half portion of the diameter OC, whereas for $x_i > AM$ it goes further beyond point C on the X axis.

Notably, a simpler equivalence emerges. The tangent linkage represents the angle pertaining to any household within the continuous bounds of angles AOP and POV”, the former for $x_i \leq AM$ and latter for $x_i > AM$, both with the constant base OP. An elaborate example is also given in the Figure 3 and notes under it.

6.4 A Modified Index PS1B on Absolute Contribution values

Notably, in the preceding index the contributions of x_i values $< AM$ are positive, and for x_i values $> AM$ become negative. Therefore, to have a positive contribution from each value we next include the absolute values of contributions by moving to the index:

$$(1/N) * \sum \text{ABS} [\{(AM-x_i) / (AM+x_i)\}].$$

Now by virtue of the denominator being $(AM+x_i)$, in aggregate, the values of x_i below AM contribute more than those above AM, resultantly the index capturing inequity (and not merely inequality *per se*).

On looking at the extreme cases, in the case of extreme concentration

when each of the (N-1) poor has zero income, and the sole rich has N*AM as income, the averaged sum of absolute contributions becomes:

$$(1/N) * [(N-1)*1 + \{(N*AM- 1*AM)/ (AM + N*AM)\}],$$

$$\text{or, } (1/N) * [(N-1) + \{(N-1)/ (1+N)\}],$$

$$\text{or, } (1/N)*(N-1)*\{1+ 1/ (N+1)\},$$

or, $(1/N)*\{1/(N+1)\} * \{N^2 +N -2\}$, which for $N \gg 1$ tends to become 1 from the lower side {technically, to set the value of extreme inequality as one, index PS1B needs a multiplicative correction factor of $\{N*(N+1)\}/(N^2 +N -2)$ }. On the other hand, for the egalitarian case of all $x_i = AM$, the contribution of each term is zero, and thus the value of the index is zero, as expected. Therefore, the value of the index has a range between 0 and 1.

Some of the characteristics of this index, in a nutshell, are that in the case of extreme inequality it tends to 1 for a large N, whereas in the case of an egalitarian society it becomes 0, therefore, it is not open-ended, Further, it holds the principles of anonymity, relative income, and population by construction. We next look at an AM preserving Pigou-Dalton transfer k from $c > AM$ to $a < AM$ that results into incomes such that $(a+ k) < AM < (c-k)$. The fall in the contribution of a transferor rich on computing the absolute values is $(c-AM) / (c+AM)$ to $(c-k -AM) / (c-k+AM)$. Further, the fall in the contribution of the receiving poor is $(AM-a) / (AM+ a)$ to $(AM-a-k) / (AM +a+k)$. Therefore, as both contributions fall the index falls and the Pigou-Dalton principle holds.

The exact change can be computed as,

$$(-2*k*AM)*[\{(1/(AM+a))\} * \{1/(AM+a+k)\} + \{(1/(c+AM-k))\} * \{1/(c+AM)\}], \text{ which is clearly a net fall.}$$

6.5 PS2 Index

We next try the index PS2 of the family by setting $\phi = 2$ as,

$PS2 = (1/N) * [\sum \{(AM-x_i)/(AM+x_i)\}^2]$, thus having square of the contributions to PS1, as its contributions.

Now again by virtue of the denominator being square of $(AM+x_i)$, against the numerator being square of $(AM-x_i)$, a value of x_i below

AM contributes more than the value above and equidistant from AM, resultantly the index captures inequity (and not merely inequality *per se*).

Taking again the case of extreme concentration, with (N-1) households having zero income and the sole rich the entire income N*AM, this index becomes,

$$(1/N) * [(N-1)*1 + \{(AM- N*AM)\}^2 / \{(AM + N*AM)\}^2]$$

$$\text{or, } (1/N) * [(N-1) + \{(N-1)\}^2 / \{(N+1)\}^2]$$

$$\text{or, } (1/N) * (N-1)* \{1/(N+1)\}^2 * [(N+1)^2 + (N-1)]$$

$$\text{or, } \{(N-1)\} * \{1/(N+1)^2\} * \{(N+3)\},$$

$$\text{or, } \{N^2 + 2N - 3\} / \{N^2 + 2N + 1\}.$$

Here the result indicates that if $N \gg 1$, the index tends to 1 from the lower side. And in the case of an egalitarian society, as all terms are equal to AM, each contribution becomes zero. Thus the index becomes zero as expected.

Some of the characteristics of this index, in brief, are that in the case of extreme inequality it tends to 1 from the lower side, whereas in case of perfect equality it becomes zero, thus it is not open-ended. It holds the principles of anonymity, relative income, and population by construction.

Let us try a Pigou-Dalton transfer k from $c > AM$ to $a < AM$ such that AM is preserved and the final incomes follow $(a+k) < AM < (c-k)$.

One can straight away, find that as a transfer from a rich to a poor brings the income of the rich closer to AM, his contribution, which is:

$\{(AM-c) / (AM+c)\}^2$ falls. Similarly, as the resultant income of the poor comes closer to AM his contribution also falls. Hence the index falls, as it gives a fall in contribution on both the poor's and rich's sides. A more formal treatment computes the exact change (new minus original) on the side of the transferee poor as:

$$\text{or, } 4*k*AM*\{(1/(AM+a)^2)\} * \{1/(AM+a+k)^2\} * \{-(AM^2-a^2) + k*a\}$$

$$\text{The expression in square brackets is } [\{-(AM-a)*(AM+a) + k*a\}]$$

Now, as $(a+k) < AM$ or $k < (AM-a)$ and in any case $a < (AM+a)$, this expression is negative indicating a fall in the value of contribution on the side of the poor. And on the rich person/ household side as the

income c falls to $(c-k)$ the contribution changes (new minus old) by,

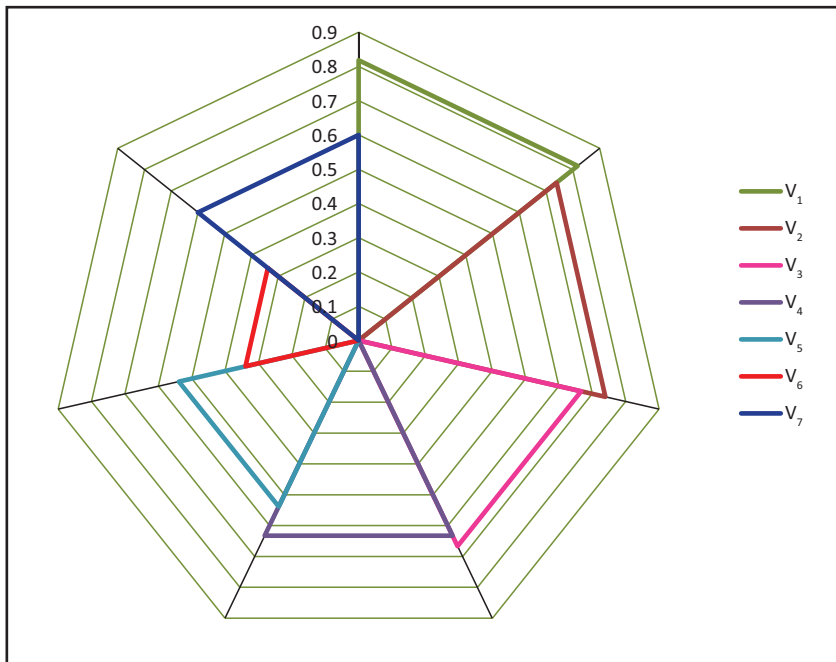
$$\text{or, } \left\{ \frac{1}{(AM+c-k)^2} \right\} * \left\{ \frac{1}{(AM+c)^2} \right\} * 4 * k * AM * [- (AM+c) * (c-AM) \} + k * c]$$

Now as $(c-k) > AM$ so $(c-AM) > k$ and $(AM+c) > c$ so the overall expression is negative indicating a fall on the rich side too. Therefore, the sum of contributions (and overall average) leads to a fall in the value of the index, on an AM preserving transfer, and thus the Pigou-Dalton criterion is fulfilled.

A PS2 Index Example

The starting point of our depiction is the 7-term example used for the PS1 Index that had led to Figure 2. As PS2 Index uses squared terms facilitating to use absolute values, the seven terms before squaring, give

Figure 4: Areas proportional to Contributions $\{(AM-x_i)/(AM+x_i)\}^2$ to PS2



Authors' compilation.

[ABS {(AM-x_i)/ (AM+x_i)}] values as 0.818, 0.739, 0.667, 0.633, 0.538, 0.339 and 0.600, labeled as V₁ to V₇ in **Figure 4**. We would in fact be using product of these terms to reach the squared values.

In order to draw a spider diagram, we take each pair of consecutive radiating arms as these terms, like the first triangle having V₁ as each of its radiating arms; the second triangle having V₂ as each of its radiating arms, where the second arm of the first triangle is along the first arm of the second triangle, and so on. Now, the first triangular area is V₁*V₁*sin(2π/N); the angle (2π/N) being in radians, which is nothing but the angle (360/N) in degrees. Hence the first seven triangular areas are proportional to (V₁)², (V₂)², (V₃)² ... and (V₇)², respectively, capturing the square components with a common multiple coming from the half of the sine value of the constant angle.

Therefore, for N being 7, the seven triangular areas in the spider diagram, starting the vertical axis and going clockwise, depict areas that are (1/2)*sin(2π/7) times of the respective contributions {(AM-x_i)/ (AM+x_i)}². On computation these contribution values are approx. 0.669, 0.546, 0.444, 0.400, 0.290, 0.115 and 0.360 respectively, averaging 0.4034, which is the value of PS2 index for this example. The triangular areas depicted in Figure 4 clearly manifest how starting with the smaller x_i terms the contributions are large and diminish towards AM, and then rise again but reaching relatively smaller values owing to the inbuilt-rising denominator components.

6.6 A Modified Index PS2B

We next try a modified index by taking the (positive) square root of the preceding index, after the final step.

Or, sqrt [(1/N)* ∑ { (AM-x_i) / (AM+x_i) }²], its value being the square root of PS2 Index which was 0.4034, gives PS2B as 0.635 for the seven-term example.

This too has the characteristics of reaching 1 from the lower side in case of extreme inequality, while reaching zero for perfect equality, thus not being open- ended. Further, by construction it holds principles of anonymity, relative income, and population.

In the case of an AM preserving Pigou-Dalton transfer k from $c > AM$ to $a < AM$ such that AM is preserved and the final incomes follow $(a + k) < AM < (c - k)$, the values of the index are nothing but positive square root values of the preceding index. As such again there is a fall in the value on the side of both poor and rich. Therefore, for such a transfer the index falls, holding the Pigou-Dalton principle.

6.7 Indices Recast into Tax Subsidy (TS) Format

Next we introduce a tax and subsidy regime linked to household incomes. For it the rule set is to tax any household with an income above AM, and to give subsidy to any household with an income below AM, leaving income exactly equal to AM as unchanged.

Further, let the combined tax-subsidy rule be such that the tax or subsidy is on the distance of the income from AM and is proportional in nature but its rate being less than 100 per cent. Therefore, the households with incomes below the AM get a subsidy in proportion to $(AM - x_i)$; and those with incomes above AM get taxed in proportion to $(x_i - AM)$.

Now let us transform the incomes $x_1, x_2, x_3, \dots, x_{(N-1)}, x_N$ to $s_1, s_2, s_3, \dots, s_{(N-1)}, s_N$ such that $s_i = (AM - x_i)/AM$, basically a transformation comprising of three elements namely, shift of axis by AM, change of direction and an AM fold contraction. Therefore, s_i is positive for $x_i < AM$, and negative for $x_i > AM$, while being zero for $x_i = AM$. As a result a positive s_i indicates eligibility to get subsidy (in the first place the basis to choose the alphabet 's'); and a negative s_i indicates applicability of a tax (a negative subsidy).

Now, we can algebraically express that if income of a household before tax is $x_i > AM$, it invites a tax, $(x_i - AM) * t$, where t is the constant proportional tax rate such that $0 < t < 1$. Here, in case of an income below AM, the tax amount algebraically being $(x_i - AM)*t$ is thus negative, automatically becoming a subsidy amount of $(AM - x_i)*t$.

It is critical to keep $t < 1$, as otherwise it may flip income levels across AM. All the more it is normatively not acceptable. For instance, the literature on 'compensation principle' to compensate for circumstances

also recognizes ‘rewards principle’ for efforts (Francisco *et al.* 2015) on distinction between the two principles (Ferreira, Francisco, H.G.; Paragine, Vito 2015).

Now as a combined tax-subsidy example, if say, $t = 0.15$, it indicates a 15 per cent tax on distance of income above AM, and a 15 per cent subsidy on distance of income below AM, where both the tax and subsidy can be captured by a single rule.

Moreover, algebraically the tax on the i^{th} income x_i being $(x_i - AM) * 0.15$, its summation over all the i terms becomes,

Total tax less subsidy over all households,

$$= \{(x_1 - AM) + (x_2 - AM) + (x_3 - AM) + (x_4 - AM) + (x_{(N-1)} - AM) + (x_N - AM)\} * t \dots (i)$$

or $\{N * AM - N * AM\} * t = 0$, here of course if the x_i 's are arranged in the ascending order, the initial terms are negative and later ones are positive enough to render the summation zero. The cherry on the cake is that the post-tax-subsidy transformation of incomes, the new AM value of income remains the same as the original one.

In fact, on multiplying and dividing equation (i) by AM one gets, the summation,

$$= (AM) * \{-s_1 - s_2 - s_3 \dots - s_{(N-1)} - s_N\} * t$$

Or thus also zero, rendering the summation $\sum s_i = 0$

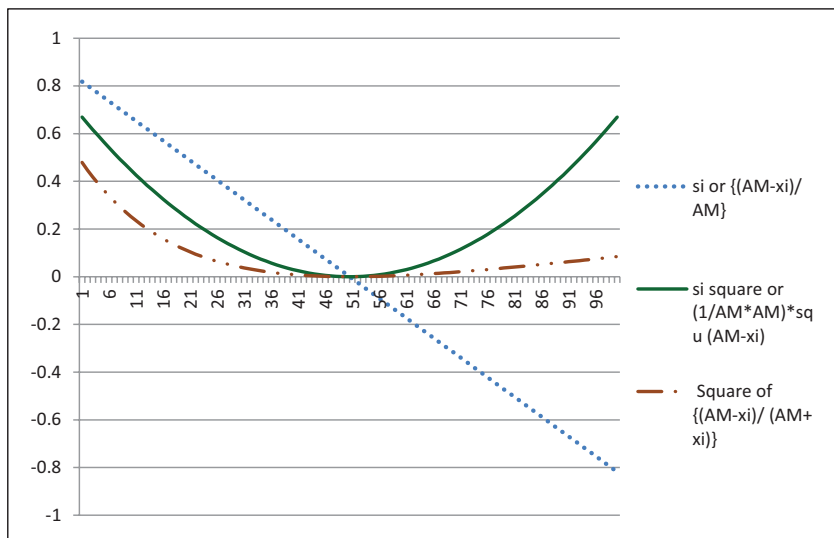
Therefore, a characteristic of such a tax and subsidy format based on the distance from AM leads to a ‘fair’ tax, in the sense that all that is collected from ‘rich’ is redistributed among ‘poor’ leaving no surplus or deficit. This underscores possibilities of self-sustaining fiscal-cum-development policies. Here, it is assumed that the costs of tax collection and subsidy distribution are zero. In real life, if electronic taxation at source and direct benefit transfer are in place, the costs can be significantly diminished, especially if the taxation machinery is collecting other taxes too and the development machinery is distributing other benefits too. Therefore, this tax-cum-subsidy format entails only minimal marginal costs.

Now, we revisit the index $[(1/N) * \sum \{(AM-x_i)^2\} / \{(AM+x_i)^2\}]^2$, along with the values of s_i and s_i^2 across various terms to have a feel of their shapes. These are manifested in **Figure 5** based on the data assumed in the SDG Target 10.1 experiment, and give an idea of how s_i changes with incomes, and resultantly while s_i^2 captures only inequality *per se*, the above index goes much further being a poverty sensitising one and captures the contributions of various terms such that the total weightage accorded to all poor ($x_i < AM$) is more than that to all rich ($x_i > AM$). In fact, the Index is alternatively expressible as averaged summation over,

$$s_i * s_i * \{AM / (AM + x_i)\}^2.$$

As an alternative expression in terms of s_i , one can algebraically express the unified tax and subsidy, as a tax of $(x_i - AM) * t$ or $(-)$ $s_i * AM * t$; where s_i itself being negative for $x_i > AM$ renders it as a positive tax, and s_i being positive for $x_i < AM$ renders it as a positive subsidy.

Figure 5: SDG Target 10.1 Experiment Based Square of $\{(AM-x_i) \text{ upon } (AM+x_i)\}$



Source: Authors' compilation.

6.8 PS Generic Index Two

Another family of indices covered in this paper is named ‘Poverty Sensitising Generic Index Two’, set as another generic family.

PS Generic Index Two = $[1/(N)] * \sum [(AM/x_i)^\phi * \{\ln(AM/x_i)^\phi\}]$, the summation is over all the i values, AM can alternatively be written as \bar{x} , and ϕ is positive, the varying values of which give specific indices of this generic family. By construction this Index assigns a higher weightage to incomes lower than AM, to sensitize better on inequity.

For simplicity the parameter ϕ is set as 1 for the first member called of PS3, which renders it as;

$$PS3 = [1/(N)] * \sum [(AM/x_i) * \{\ln(AM/x_i)\}]$$

We recall that Theil-T and Theil-L Indices have $(x_i/AM) * \ln(x_i/AM)$ and $\ln(AM/x_i)$ as the respective building blocks, which are then averaged over the number of terms. For the PS3 the building block set here is $(AM/x_i) * \ln(AM/x_i)$.

Notably, here if all terms are equal, each (AM/x_i) becomes 1, leading the value of the Index to Zero, as expected of an inequality index.

Here the aim is to properly sensitize on the plight of the ‘poor’, by capturing it to the extent possible in the index right from the stage of a member’s contribution. This necessitates drilling down to the contributions coming from different percentiles.

Notably, in 2011-12 the rural poverty line in India was Rs. 816 per capita per month, while the average MPCE (AM) was Rs. 1,430 per capita per month, being 1.7525 times of the poverty line. Therefore, the poverty line was at $(AM/1.7525)$ and thus half of the Poverty Line was at $(AM/3.5050)$. As a result, the ratio of the value of contribution to the PS3 index by two individuals at these levels was $(3.5050) * \ln(3.5050)$ to $(1.7525) * \ln(1.7525)$, or 4.3959 to 0.98323 or 4.47 i.e. around 4.5 times. This looks high enough to opt for this index with $\phi = 1$ as above, though to accommodate for higher (lower) degrees of poverty sensitization a higher (lower) value can be assigned to ϕ . For instance, a choice of $\phi = 2$ raises the ratio to 8.94, whereas a choice of $\phi = 0.5$ reduces it to 3.16.

6.9 Theil-T and PS3 narrative

Theil-T is $(1/N)$ of $\sum (x_i/AM) \ln (x_i/AM)$

or, $(1/N)* \sum [\ln \{(x_i/AM)^{(x_i/AM)}\}]$

or $(1/N)* [\ln \{\prod (x_i/AM)^{(x_i/AM)}\}]$ -----(1) [On taking N inside, it would in fact be the ln of the Nth root of the expression within the curly brackets]

Now, PS3 is $(1/N)$ of $\sum (AM/x_i) \ln (AM/x_i)$

or $(1/N)* \sum [\ln (AM/x_i)^{(AM/x_i)}]$

or $(1/N)* [\ln \{\prod (AM/x_i)^{(AM/x_i)}\}]$ -----(2) [Similarly, on taking N inside, it would thus be the ln of the Nth root of this expression within the curly brackets]

Now, as an example, if x_1/AM , x_2/AM and x_3/AM are 0.5, 1 and 1.5; thus x_1 is ‘poor’, x_2 is at the AM and x_3 is ‘rich’. It leads to the values of Theil-T, Theil-L and PS3 indices as 0.087, 0.096 and 0.372 respectively, indicating PS3 to be significantly more poverty sensitising, in this case.

Limits of the Index PS3

By virtue of the (AM/x_i) weightage term in it, this index remains open-ended towards the lower end, as x_i approaches zero. In general, some households may have zero incomes, but not zero MPCE, due to the transfer channels of safety nets and social assistance etc. Therefore, in real life situations, at the household level MPCE can be opted over PCI, or household consumption expenditure over the household income.

Nevertheless, a case of high inequity when say, just 1 ‘rich’ household captures $(N-1)*AM$ income, and leaves only $1*AM$ to be shared amongst the rest $(N-1)$ ‘poor’, leads to each ‘poor’ getting only $\{AM/ (N-1)\}$ as income.

This leads to the value of the index as,

$$PS3 = (1/N)* [\{(N-1)*(N-1)*\ln(N-1)\} + \{1/(N-1)\}*\{\ln\{1/(N-1)\}\}]$$

$$\text{or} = (1/N)* [\{(N-1)*(N-1)*\ln(N-1)\} - \{1/(N-1)\}*\{\ln(N-1)\}]$$

$$= (1/N)* [\{(N-1)^2\} - \{1/(N-1)\}]*\{\ln(N-1)\}$$

This for $N \gg 1$ approaches (from below) $N*(\ln N)$.

By construction the index clears the tests of Anonymity, Population and Relative income principles.

Now an AM preserving transfer k from a rich having an income 'c' to poor having an income 'a' leads to two changes:

Contribution of the transferee poor changes to $\{AM/(a+k)\} * \ln\{(AM/(a+k))\}$ from $(AM/a) * \ln(AM/a)$, where $a < (a+k) < AM$. And the contribution of the transferor rich changes to $\{AM/(c-k)\} * \ln\{(AM/(c-k))\}$ from $(AM/c) * \ln(AM/c)$, where $AM < c-k < c$.

Next, for a generic contribution term $(AM/x) * \ln(AM/x)$, on taking its partial differential for a small change in x , and equating it to zero, the first order condition (FOC) leads to $x=e*AM$ as an optimum. The second order condition (SOC) brings out that it is a minimum.

On the side of the poor, as x increases from zero towards AM , its contribution monotonically falls. On the side of the rich the contribution is invariably negative and as x increases, it initially keeps algebraically falling till x reaches $e*AM$ and thereafter rises. Hence, on the side of the poor there is invariably a fall in the value of the contribution. However, on the side of the rich the fall (value being negative) is upto $e*AM$ followed by a rise towards zero. Notably, as the income of the rich transferor falls due to transfer, so between the AM and $e*AM$ portion, his contribution (algebraically) rises (but any rise computed is less than the fall on the side of any poor). Further, for the entire portion $x_i > e*AM$ towards infinity, on transfer, the contribution falls. In a nutshell, the value of the index invariably falls, holding Pigou-Dalton principle for the PS3 Index.

Now, as x tends to infinity, and since $(AM/x) * \ln(AM/x)$ can be rewritten as $y * \ln(y)$ by using $y = (AM/x)$; y tends to zero. The expression $y * \ln(y)$, again rewritten as $\ln(y) / (y^{-1})$, on application of L'Hopital's rule, by differentiating both numerator and denominator, becomes $(1/y)$ upon $(-1/y^2)$. By setting $y = (0 + h)$, where h is a small positive infinitesimal value, the expression becomes $(-) h$, when x tends to infinity. The contribution thus keeps algebraically rising beyond $x=e*AM$ and tends to zero from below asymptotically.

Expressed in numbers, as x_i exceeds AM the contribution becomes negative and keeps initially algebraically falling till x reaches $e*AM$ (or

approx. $2.718 \cdot AM$), where it reaches down to the value $(1/e) \cdot \ln(1/e)$ or $(-1/e)$ or $(-) 0.368$. The contribution thereafter starts algebraically rising from $(-1/e)$ or $(-) 0.368$ towards 0 asymptotically, for instance for the $x=10 \cdot AM$ and $x = 100 \cdot AM$ terms, the contributions to PS3 becomes $0.1 \cdot (\ln 0.1)$ i.e. (-0.230) and further algebraically rises to $0.01 \cdot (\ln 0.01)$ i.e. (-0.0461) respectively, overall value of the index remaining positive thanks to the terms below AM .

It is worth noting that, for the SDG Target 10.1 experiment, the initial value of the index PS3 is 0.817 which falls to 0.686 after the 5-year income-converging growth rates, giving a perceptible fall of about 16 per cent over this period.

6.10 A Sensitivity example on PS1 PS2 PS3 and Atkinson Index

In our SDG 10.1 original experiment, we introduce a change that the richest person's income doubles from Rs. 1,000 to Rs. 2,000, but without income of any of the other 99 persons changing, a case of deepening inequalities, Now in line with the increased inequality, the new values of PS1, PS2 and PS3 indices on computation (with a higher mean of Rs. 560 instead of Rs. 550) increase by 8.64, 3.92 and 5.01 percent respectively of their original values. In comparison, the value of Atkinson index rises by 2.26 per cent, in the case of the risk aversion parameter set at 3.

7. Inequity Augmented Lorenz Curve (based on the distribution revealed ratio) and Poverty Sensitising Index 4 (PS4)

7.1 Gini Coefficient revisited from *ab initio* and Lorenz Curve

As an expression linked to Lorenz curve, Gini coefficient is mathematically defined as half of the relative mean value difference, i.e. half of mean absolute difference of all pairs of items of the attribute (income here) normalized by average value of the attribute. The mean (or total) is assumed non-negative, usually positive, to rule out (value of) a Gini coefficient falling outside the closed interval $[0,1]$. So across N persons if X_i and X_j are incomes of the i^{th} and j^{th} person respectively, and AM is the average income (assumed positive) and x_i and x_j are X_i

and X_j divided by AM to normalise; the Gini coefficient is expressed as:

$$\{ \sum \sum |x_i - x_j| \} / \{ (2 * N^2) \},$$

where, the double summations are over all the pairs of i's and j's from 1 to N each (including all $i = j$ though contribution for self-pairing collapses to zero for this expression). For continuous values, the corresponding double integral expresses it.

As an example of extreme inequality, if an island has N households of which (N-1) poor earn zero income (subsisting on social assistance) and the sole rich therefore earns $100 * AM$, where AM is positive, the Gini coefficient can be computed to be as $(N-1) / N$. In the case of a whisker less yet very high inequality, when income $1 * AM$ is equally shared by all the (N-1) 'poor' each getting a mere $\{AM / (N-1)\}$ and the sole 'rich' getting $(N-1) * AM$, the Gini coefficient becomes $(N-2) / N$.

7.2 Gini Coefficient and Lorenz Curve- Some Stylised Facts

Gini coefficient captures inequality *per se* but not inequity, as it does not accord higher weightage to absolute differences below AM, compared to similar ones above AM.

It ranges between 0, for an egalitarian society having no inequality; to 1 for the most unequal distribution when theoretically just 1 person earns the entire income and the number of persons is very large. Literature has analysis of negative incomes and Gini coefficients falling outside $[0,1]$ needing correcting factors, but we would stick to positive (non-negative to be precise) terms and a positive AM (at least one term is positive even when all others are zero).

Multiplication (or division) of incomes by an identical (positive) number leaves the value of Gini coefficient unaltered, as now the normalization is by the AM also multiplied by such number.

Identical positive addition to each income reduces proportions and thus diminishes the value of Gini coefficient. By contrast, subtraction by an identical number increases its value so long as each income remains non-negative so that the Gini coefficient remains within bounds $[0,1]$.

By definition Gini Coefficient holds the principles of anonymity, population and relative income. On a Pigou-Dalton regressive transfer

of k from a poor having income ‘ a ’ to a rich having income ‘ c ’, where each of a , c and $k > 0$ and $a < AM < c$, the two new incomes become $(a-k)$ and $(c+k)$. The Lorenz curve therefore, moves downwards from $(a-k)$, and remains downwards till $(c+k)$ is reached, leading to a higher area of the lens and the value of the resultant Gini coefficient. One may be rightfully tempted to recheck it algebraically. Here out of the $N*N$ matrix the intra-contributions from all $(N-2)$ numbers other than ‘ a ’ and ‘ c ’ forming an $(N-2)*(N-2)$ matrix remain unchanged. Now if n_l is the number of elements lower than ‘ a ’; then out of the N pairs of ‘ a ’, the new contributions from n_l fall by k each, whereas for $(N-2-n_l)$ rise by k each, while for one (the c) rises by $2k$ and from itself of course remains 0 {unchanged from $(a-a)$ to $(a-k) - (a-k)$ }. Thus, the rise for pairs of ‘ a ’ adds up in terms of k to $(N - 2*n_l)$. Similarly, if n_h is the number of elements higher than ‘ c ’, their contributions rise in terms of k by $(N-2*n_h)$. In totality, the rise in contributions in terms of k becomes $(2*N - 2*n_l - 2*n_h)$ or $2*(N - n_l - n_h)$ which is invariably positive as $N > (n_l + n_h)$. Therefore, the Gini coefficient rises holding the Pigou-Dalton transfer principle.

7.3 Traditional and Inequity Augmented Lorenz Curves using PS4

Inequity Capturing Operations

The traditional Lorenz curve captures inequality *per se* but not inequity. Therefore, an ‘Inequity Augmented Lorenz Curve’ is proposed based on distribution revealed ratio capturing an element of inequity between the ‘poor’ and ‘rich’ using these terms again to relatively denote household incomes below and above the Arithmetic Mean (AM). For it a parameter denoted as ‘ r ’ being the ratio as **revealed by the given distribution** is used, which is defined here as B upon U i.e. B/U where,

- i. $B = \{($ the sum of all terms **below** AM) + (**half** of any term(s) exactly equal to AM) $\}$ and
- ii. $U = \{($ the sum of all terms **above** AM) + **half** of any term(s) exactly equal to AM) $\}$

Therefore, smaller this ratio, larger is the inequity. We can also term $(1-r)$ as ‘ s ’ and call it as the ‘inequity gap ratio’. Therefore, in the

case of an egalitarian society ‘r’ would be unity and thus ‘s’ would be zero. On the other hand, for extreme inequality ‘r’ would be zero and thus s would be unity.

Once ‘r’ is computed we proceed as follows:

- i. Initially, we make a progressive transfer proportional to ‘s’ ($0 \leq s \leq 1$) from ‘rich’ to ‘poor’ so that any x_i becomes $x_i + s*(AM-x_i)$ or say, x_{ip} . This in essence acts as a subsidy to each poor and a negative subsidy i.e. a tax on each ‘rich’, leaving unaltered the incomes that are exactly equal to AM. Notably, the sum of new terms becomes:

$$\Sigma x_{ip} = \Sigma x_i + s*(AM-x_i) \text{ over } i \text{ varying from } 1 \text{ to } N,$$

$$\text{or } N*AM + s*(N*AM - N*AM),$$

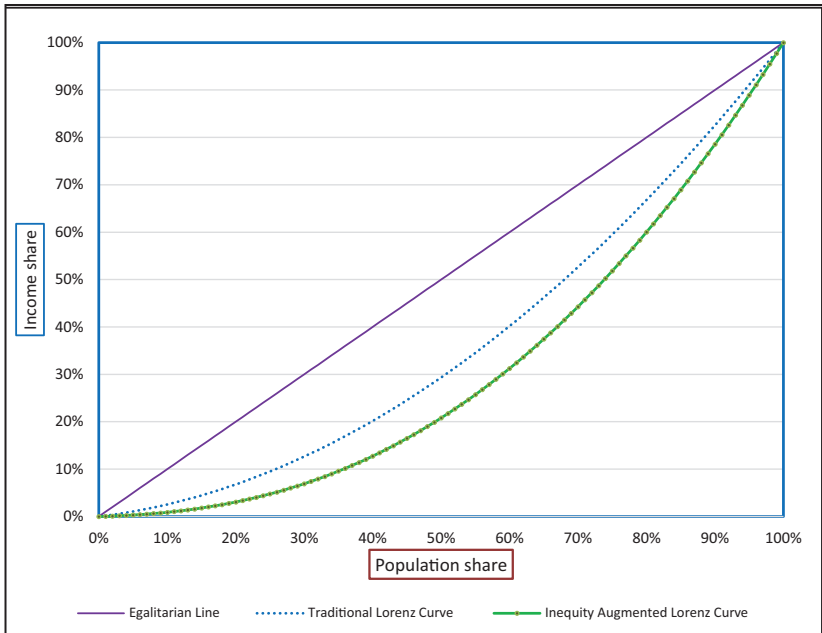
or $N*AM$ and thus this is an AM-preserving operation.

- ii. Now, this progressive transfer is akin to an equitable proportional subsidy for ‘poor’ and tax on ‘rich’ as designed with distances from AM, thus each x_i changes by a varying multiplicative term v_i i.e. $x_i*v_i = x_{ip}$ and each x_i has its specific corresponding v_i , while v_i always remains positive (for all $s < 1$). Among the ‘poor’ it stretches incomes in the lowest deciles by larger values of v_i , whereas among deciles below and closer to AM it stretches incomes by smaller values of v_i . For example, we take the case of s being 0.5, thus for x_i as $(AM/4)$, x_{ip} becomes $(AM/4) + 0.5*\{(AM)-(AM/4)\}$ or $\{(5*AM)/8\}$ and so its v_i is $\{(5*AM)/8\} / (AM/4)$ or $5/2$; and for x_i being $(3*AM/4)$ x_{ip} becomes $(3*AM/4) + 0.5*\{(AM)-(3*AM/4)\}$ or $(7*AM/8)$, and its v_i is $7/6$. Further, among ‘rich’ as s is 0.5, for x_i $(5*AM/4)$, $x_{ip} = (9*AM/8)$ its v_i is $9/10$, and for x_i being $(7*AM/4)$, x_{ip} is $(11*AM/8)$ and its v_i is $11/14$. Thus, among ‘rich’ it diminishes the incomes in the highest deciles by larger values of v_i , whereas it diminishes deciles above and closer to AM by smaller values of v_i . It thus acts as a differential contraction towards AM, remaining hinged to it.
- iii. This progressive operation thus shrinks income gaps and so diminishes the value of Gini coefficient. By contrast one needs to rather stretch income gaps to augment value of the coefficient, for which we multiply each x_i by inverse of its corresponding v_i to attain

x_{ia} and draw augmented Lorenz Curve based on these x_{ia} values. This operation acts as a differential amplification from AM and remaining hinged to it.

- iv. Notably, $(x_{ia}/x_i) = 1/v_i$, where, $(x_{ip}/x_i) = v_i$ implying that geometric mean (GM) of x_{ip} and x_{ia} is x_i . As the resultant distribution of x_{ia} augments the income gaps its Gini Coefficient is higher than that of the traditional Lorenz curve.
- v. As an example, we revisit the 100-term SDG 10.1 experiment. For it the value of inequity capturing parameter ‘r’ comes to approximately 0.415 implying that an average ‘poor’ has only about 41.5 per cent of income of an average ‘rich’. Thus, the value of ‘s’ is approximately 0.585, and in the first step the subsidy/ tax is equivalent to approximately 58.5 per cent of distance from AM.

Figure 6: Traditional Lorenz Curve and Inequity Augmented Lorenz Curve Capturing the Distribution Revealed Ratio



Source: Authors’ compilation.

The Gini coefficient of the traditional Lorenz Curve, which was 0.275, rises by approximately 40.4 per cent to 0.386 for the ‘Inequity Augmented Lorenz Curve’ both depicted in **Figure 6**.

- vi. For an egalitarian society with each of the N households having an income AM, the value of r is 1. Therefore the value of s is zero, each v_i becomes $v=1$ and the Traditional Lorenz Curve remains intact without any augmentation.
- vii. In the light of the preceding analysis, the building blocks of this poverty sensitising index, that we term PS4, are:

$$[1/\{x_i+s*(AM-x_i)\}]*x_i,$$

where the inequity gap ratio $s = (1-r)$ and

the inequity capturing ratio $r = \{(\text{sum of incomes below AM} + \text{half of incomes exactly AM}) / (\text{sum of incomes above AM} + \text{half of incomes exactly AM})\}$.

On application of the PS4 operations, revealed by the distribution, one can reach the ‘Inequity Augmented Lorenz Curve’, and as already stated Figure 6 depicts it for the SDG 10.1 experiment, alongside the Traditional Lorenz curve.

8. Inequality *per se* Capturing Indicators

The Variance, Standard Deviation (SD), Coefficient of Variation (CV) capture only inequality *per se*, *sans* inequity. All these are, of course, anchored to Arithmetic Mean (AM). Moving a bit away from the base anchored to AM, an index can be conceived with its base anchored to Geometric Mean (GM), but with a word of caution that even if one term is zero, GM collapses to zero, so it can be used for say, MPCEs but not incomes. For this purpose value of the proportion (x_i / GM) for an x_i can be viewed as a stretch or shrink operation upon GM, depending upon whether $x_i > GM$ or $x_i < GM$ unless $x_i = GM$. Mathematically we may use the term ‘stretch’ to encompass all the three cases, it being > 1 , or < 1 or $= 1$. Such a value of stretch can easily be used to evolve an index.

A problem encountered with the stretch operations on GM is that the product of all stretches is not much consequential being unity by

definition. On taking ‘ln’ of individual stretch values one gets positive numbers for a stretch > 1, negative for < 1 and 0 for = 1. The sum of all ‘ln of stretch’ values’ accordingly adds up to zero, therefore by itself failing to capture inequality. Same is the story with the reciprocal of stretches GM/x_i values, only with a change that their geometric mean, say, $GM_{\text{reciprocal}}$ being $(1/GM)$ of original value.

Nevertheless, on squaring up the ‘ln’ of reciprocal-stretch results (taking a cue from the handicap of sum of deviations from arithmetic mean being zero, necessitating to square up to capture variations), one turns each of these contributions into a positive value (non-negative to be precise), which can be similarly added up and averaged upon to capture inequality.

Therefore, the averaged squared ‘ln of GM’s reciprocal-stretches’ can be an index of **inequality** expressed as:

$$\left[\frac{1}{N} \sum \{ \ln(GM/x_i) \}^2 \right]$$

Moreover, the value of this index remains unchanged when each element is replaced by its reciprocal, as the new GM is reciprocal of the original, and the index essentially captures the relative ratios of elements.

As an example, three values 4 and 6 and 9 give GM as 6, so the (GM/x_i) terms are $2/3$, 1 and $3/2$ giving $\ln(GM/x_i)$ values as $\ln(2/3)$, $\ln 1$ and $\ln(3/2)$ or being $(-)$ 0.4055, 0 and $(+)$ 0.4055 (adding to zero by definition), which on being squared give contributions as 0.1644, 0 and 0.1644 respectively, and on averaging the value of the index as 0.1096. Obviously, the contributions aren’t poverty sensitising, and thus capture only inequality *per se*. Further, the reciprocal values $1/4$ and $1/6$ and $1/9$ give new geometric mean as $1/6$, and so the new geometric mean divided by each term again gives as $2/3$, 1 and $3/2$, leading to an identical value of index.

Moving further to different data sets but having the same GM, as an example one may select three data sets first 6, 6 and 6; second, 4, 6 and 9 and third 2, 6 and 18 each having 6 as the GM. The averaged sum of squared ‘ln’ values in the first case is 0, in the second case (as in the preceding para) it increases to 0.1096 and in the third case 0.8096

(here, $\ln(3)$ being 1.0986, $\ln 1$ being 0 and $\ln(1/3)$ being minus 1.0986; on squaring give, 1.2069, 0 and 1.2069, which on averaging give 0.8046). it shoots up to 0.8046, thus capturing increasing inequality.

An important property expected of a purely inequality *per se* capturing index is that it is neither biased towards small values nor the bigger values. For example let a data set comprise of 5 (prime) values namely 11; 11; 11; 11 and 37. Let another data set comprise of, different (yet linked) five values 37; 37; 37; 37 and 11. The index used can be treated as richness-poverty neutral if its value is identical for both the data sets, which in fact is found so for this index being 0.235. On generalization, a data set of N values comprised of $(N-1)$ poor persons each having an income of x units and just 1 rich person having an income of $k*x$ units (where k is positive and large) is expected to have the same value of this index as another data set of N values comprising of just 1 poor person having an income of x units and $(N-1)$ rich persons each having income of $k*x$ units, although the GM of the new data set would differ.

The index becomes zero when all values in the data set are equal, leading to GM also the same, and thus each \ln contribution being zero. Therefore, minimum value of the index is zero, correctly capturing zero inequality.

A further look at this index reveals that it follows Anonymity, Population and Relative income principles, by construction. Another beauty of this index is that by virtue of squaring up negative values, the index comprises of equal contributions from a stretch and its reciprocal stretch, making it symmetric, which an index purely to measure inequality *per se* is desirable to be.

A progressive transfer preserving GM

Let us next analyze it for any GM preserving Pigou-Dalton transfer, from a value above GM to a value below GM, such that even after the transfer both remain on their initial sides of the GM.-

For analyzing a progressive transfer, we initially assume a 3-data set having positive values a , b and c arranged in the ascending order $a <$

$b < c$, such that $b^2 = a * c$ giving b as their GM (precisely being the cube root of $a * GM * c$). Let us take out an amount k from rich c , while add another amount k' to poor a , where k and k' are positive, such that these are GM preserving operations, while retaining $(c-k)$ and $(a+k')$ above and below GM respectively.

So $(a+k') * (c-k) = b^2$, and as $b^2 = a * c$;

so $(-a) * k + k' * (c-k) = 0$ or $(k'/k) = (a) / (c-k)$,

And as $(c-k)$ remains $> a$, therefore, $k' < k$.

Hence, one needs to add even less than k to a , to preserve the GM. This can be extended to a case of N data set, as the remaining $(N-2)$ numbers remain unaltered, and their contribution (in fact of each) on square of \ln remains unchanged, while only the transferor and transferee values undergo changes and their reciprocal-stretches and thus the two squares of \ln values fall. As this leads to fall in the value of the index, it upholds Pigou-Dalton principle.

This index ‘ $[(1/N) * \sum \{\ln(GM/x_i)\}^2]$ ’, being akin to the variance (of \ln of reciprocal-stretch), albeit in the GM realm, captures only **inequality *per se*, sans inequity**. Notably, contributions to the expression are identical for a term which is say, five-fold of GM and another term which is one-fifth of GM. Therefore, it is an inequality *per se* capturing index, symmetric in the mirror of GM for data values which are multiples or fractions of its value measured in GM terms.

The index has no upper bound, similar to Theil–L index, and for the identical reason that a very low x_i term leads to an infinitely high $\ln(GM/x_i)$ value. In practice we may make certain assumptions restricting extreme smallness of the lower terms, like use of household MPCE instead of income. Further, one can find an upper limit of the value of the index, if extreme inequality is set so as to assign out of the entire MPCE, fraction of just $1 * GM$ equally (some equality at last though of poverty distribution among poor) to each of the $(N-1)$ low income households, each getting $GM/(N-1)$, with product of their MPCEs thus being $(GM)^{(N-1)}/(N-1)^{(N-1)}$.

Now, to preserve the product of all the N values of MPCEs as (GM)^N the sole rich household needs to be assigned an MPCE of as high as GM*(N-1)^(N-1).

This theoretically results in a devastatingly high rich to (each) poor MPCE ratio of,

$$(N-1)*(N-1)^{(N-1)}$$

The contribution of each poor household to the Index is:

square of $[\ln\{GM\} / \{(GM)/(N-1)\}]$,

or square of $\{\ln(N-1)\}$,

or $\{\ln(N-1)\}^2$

and thus of all (N-1) poor households is $(N-1)*\{\ln(N-1)\}^2 \dots\dots(i)$

And the contribution of the sole rich is square of $\ln[GM/$

$\{GM*(N-1)(N-1)\}]$,

or as squaring removes negative sign, it becomes $[(N-1)*\ln(N-1)]^2$

or $(N-1)^{2*}\{\ln(N-1)\}^2 \dots\dots\dots(ii)$

or the contribution of all the N terms, being sum of (i) and (ii) is,

or $(N-1)*\{\ln(N-1)\}^2* \{1+(N-1)\}$,

or $N*(N-1)*\{\ln(N-1)\}^2$

This on averaging over N yields $(N-1)*\{\ln(N-1)\}^2$

So as $N \gg 1$, it tends to $N*\{\ln(N)\}^2$

A revisit to the original 100-household distribution SDG Target 10.1 experiment gives the initial value of the GM-based inequality index as 0.3571, which after the 5-year converging income growths falls to 0.3108, registering a fall of 12.97 per cent say 13 per cent.

Of course the inequality *per se* capturing index,

$[(1/N)*\sum\{\ln(GM/x_i)\}^2]$ can be modified to capture inequity also if the smaller terms are assigned a higher weight, by opting for reciprocal of terms as weight. Such a reciprocal-stretch weighted version of the index can be:

$$[(1/N)*\sum(GM/x_i)\{\ln(GM/x_i)\}^2]$$

B. Universality of the Equally Distributed Equivalent (ede) Income Tool

Before summing up, it is relevant to highlight the ede as a tool, rooted in the diminishing marginal utility of income. On a revisit to the 7-term PS2 example, where PS2 is $(1/7) * \sum \{(AM-x_i) / (AM+x_i)\}^2$, with the application of utility to start with as say, power (2/3) of income i.e. $u_i = (x_i)^{2/3}$, one can compute the corresponding ede. This income (below AM) if available to each, spares some income out of total income, after such equal distribution, while keeping total utility intact. On computation in this example ede is found to be $0.74998 * AM$ or approximately $0.75 * AM$. In its parlance like Atkinson index, this gives a leeway of sparing about 25 per cent incomes keeping total utility intact. In fact, for any concave utility function such equivalence can be computed, notwithstanding the index used. As an example, on choice of PS2 as the index, one can compute such a particular value of k that results into the same value of index as calculated for PS2. This results into a tighter (smaller) value of k and thus sparing more income. This value is computed as $k=0.43$ i.e. utility $u_i = (x_i)^{0.43}$ leading to average utility for the 7-terms as 0.8009, so y_{ede} is $(0.8009)^{(1/0.43)}$ or 0.5967 and thus index as $1 - 0.5967$ or 0.4033, in order to reach back to the value of the PS2 index computed for the 7-term example in an earlier Section. In a nutshell, so far as utility function is concerned, for the 7-term example the PS2 is in sync with $u_i = (x_i)^k$ when k is 0.43. In fact, one can ascertain ede for any index, because of the underlying reason of ability to spare some income to keep utility intact in the range where the utility function is concave towards x-axis (income axis), and increasing albeit at diminishing pace, yielding its first derivative as positive and second derivative as negative.

9. Conclusions and Way Forward

9.1 Conclusions

Policy framework to reduce economic inequalities, visualises the problem from the egalitarian lens. Resultant policy advocacy is often woven around benevolence and posited in the normative strand of economics. In the process many policy responses are biased in the sense that these

focus more on assuming that one section of society needs to be taxed to subsidize another, overlooking the vast potential that remains unlocked among poor due to lack of opportunities and access. Once internalised in the narrative, the poor starting from a narrower base, can give much bigger push towards development and sustainability, while making the best out of the transfer receipts till they harness their full potential, neutralizing the handicap of circumstances.

SDG Target 10.1 rightly advocates this narrative by aiming that the income growth of the poorest 40 per cent of the population should be more rapid than the national average. In fact, efforts towards equality may be desirable on some parameters like wealth, income, consumption, income growth rates. However, inequality in itself termed in this discussion paper as inequality *per se* is not always a negative attribute; for instance a consumer desires to exercise a choice out of variety of goods and services, to boost his utility attributable to his different taste and preference. Globally people strive to conserve parameters like bio-diversity shunning away from situations like extinction of species or of monoculture. But so far as inequality *per se* is concerned, one can analyse it purely from the statistical angle of variability or lack of central tendency/ concentration. On the policy plank, inequity can't be analysed without internalising the normative values, societal ethos and humanitarian concerns.

SDG Target 10.1 underscores that income growth be normative so as to by 2030, progressively achieve and sustain income growth of the bottom 40 per cent of the population at a rate higher than the national average. Obviously, in most of the developing countries the bottom 40 per cent is large enough a population to cover all people below the poverty line, assuming that the yardstick to measure poverty is fair and applied on merits. Notably, some of the indices commonly used to measure inequality actually capture only inequality *per se* overlooking inequity. In this setting, the contribution to the Index from the population below say, arithmetic mean (AM) should be higher, than the one from above it, to make an index what is termed in the paper as poverty-sensitising (PS).

Among the prevalent inequality indices, Lorenz curve depicting cumulative income of cumulative populations, measures inequality expressed as Gini coefficient. In reality, a Lorenz curve throws much information than its uni-dimensional Gini coefficient, revealed through the curvature it takes and its slope-percentile relationship highlighting marginal incomes. For instance, its point of half the slope of the unit slope egalitarian curve reveals the percentile having half the mean income. Variance, Standard deviation, CV, etc. are anchored to AM but measure only inequality *per se* by giving equal weightage to the terms equi-distant from the AM on the side of ‘poor’ and ‘rich’ defined as those having incomes less or more than the AM respectively. A category of indices anchored to Geometric Mean (GM) is also analysed of which squared natural log of reciprocal-stretch from GM captures inequality *per se*. The Theil indices measuring inequality are anchored to arithmetic mean (AM), and in this class Theil-T index assigns a higher weightage to incomes above AM, and Theil-L index to incomes below AM. In fact, Theil-L index is anchored to GM also besides AM, as its genesis emanates from the fact that AM exceeds GM when any inequality exists. Atkinson index captures inequity through equally distributed equivalent (EDE) income, once inequality aversion parameter is specified. This paper also puts forth two generic families of poverty sensitising indices, and shows how members of these families hold the principles of anonymity, population principle, relative income principle and the Pigou-Dalton transfer principle, giving three PS indices. Based on the distribution revealed ratio the Paper also gives ‘Inequality Augmented Lorenz Curve’ as the fourth PS index.

9.2 Way Forward

In line with the essence of SDG Target 10.1, in order that the four bottom deciles achieve a higher than national average growth rate of income they need to be accepted as equal partners in this endeavor, in order to achieve and sustain such converging incomes. Given that in some attributes poor require development of their human capital; the scope of public policies should be enlarged from safety nets and social assistance to their employability and opportunities of employment. Policies adopted,

while harnessing potential of poor and internalizing their capabilities to bring them in the economic mainstream, should keep automatically opening safety nets ready. Moreover, the progress of lower deciles should be monitored through the indices that are more equity centric than merely equality *per se* centric. Such poverty sensitising (PS) indices should be used more profusely to monitor and end poverty. Further the two families of generic indices give three indices named PS1, PS2 and PS3, alongwith the modified ones PS1B and PS2B, inbuilt with a higher contribution from the population below average income than above it, are put forth to enlarge the bouquet of PS indices. Further the SDG Target 10.1 experiment analysed on the equal income increasing percentiles leading to a ten to one ratio between the highest and the lowest incomes can be suitably enlarged. For it, to attain this target various economies may evolve similar localized experiments and put the progress in the crucible of one or more PS indices to follow the trajectory. Similar to the tax-subsidy rule covered, that automatically emerges fair in nature by virtue of its linkage to PS1, various economies may also evolve such self-sustaining fiscal-cum-development policies, in the spirit that no one is left behind.

In addition to the UN bodies, the task to reduce inequalities needs more intense traction at various fora like G20. In fact, G20 summit declarations recognise the risk posed by inequalities to social cohesion, and express commitment to reduce inequalities in line with the G20 policy principles to ensure access to adequate social protection for all, to realize a society where all individuals can make use of their full potential. Towards it a comprehensive and balanced set of economic, financial, labour, health, education and social policies is visualised, including early child development (ECD), as means of building human capital to break the cycle of intergenerational and structural poverty. Further, to avert deepening of digital divide, G20 commits to equally share benefits of digitalisation, and has framed the G20 high-level policy guidelines on digital financial Inclusion through the global partnership for financial inclusion (GPMI), which need faster implementation. G20 also needs to promote collaboration on data especially, the data for development.

Therefore, as a stepping stone towards equity, to make poor digitally literate their financial inclusion in social assistance and safety net ecosystems should be a top priority to fructify achievement of not only SDG Target 10.1 but the entire Goal 10 and all the 17 Goals. In the endeavor to impart lasting economic strength to poor, right to skilling opportunities should also be enunciated and put in place with a skill enabling ecosystem. Moreover, social assistance and safety net policies should be inbuilt to be of relatively much value to the four bottom deciles. Lastly, besides the income and consumption a society needs to reduce vast wealth disparities to ensure that once an iniquitous chasm is bridged, a new one is not created.

Endnotes

- ¹ <http://legislative.gov.in/sites/default/files/coi-4March2016.pdf> last accessed on 26th August 2019.
- ² It includes caveats like measurement errors due to factors including tax evasion.
- ³ <https://wid.world/country/india/> last accessed on 19th October 2021.
- ⁴ Here words rich and poor indicate categories above and below arithmetic mean.
- ⁵ If two percentiles have an identical share of income, their ordering would be non-decreasing.
- ⁶ Park Ji-Won; and Kim, Chae Un (2021), ‘Getting to a feasible income equality’, PLoS ONE 16(3):e0249204. <https://doi.org/10.1371/journal.pone.0249204> last accessed 11th July 2021.
- ⁷ Precisely twenty six degrees thirty four minutes (26.567 degrees) and fourteen degrees two minutes (14.033 degrees) being the values of tangent inverse of 0.5 and 0.25 respectively.
- ⁸ The biased estimators use N the sample size in the denominator for division, whereas unbiased estimators apply Bessel’s correction and thus use (N-1) i.e. in this case the degrees of freedom in the denominator for the division.
- ⁹ Named after the French mathematician L’Hopital’s, and applied once or more till limit of a derivative of a function may be reached, if limit of the function is indeterminate.
- ¹⁰ To be precise fifty six minutes.
- ¹¹ A way out could be to add 1 to each term (after expressing these much above 1 by change of units), compute GM’ from these modified terms and subtract 1 to get (approximate) GM from it.

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